Everyday calculus teaching

Torbjörn Lundh

This is a collection of different citations and my own naïve pedagogical reasoning, as a mathematics teacher in higher education, concerning the teaching of mathematics at an introductory university level. I will start with some comments about the present calculus-teaching situation, from both a teacher and a student viewpoint. Then I will present an interview with one of my favorite teachers. I will also comment on some books and texts that I found helpful to me, when trying to better understand the current calculus teaching challenges. Finally, I will briefly discuss two challenges didactics research are facing today: How to manifest itself as an accepted science, and how to convince active teachers about its usefulness in daily teaching.

How is my own calculus-teaching situation? It is far from steady state. On some days, I can almost picture myself as a flexible and perceptive musician in a band. Improvising, answering and commenting the other musicians' previous phrases and musical ideas. The whole mesh of ideas that creates the calculus we know today is slick, well formulated, and, above all useful. When I am able to help students see some of its beauty, which sometimes can even be viewed in poetic terms, I feel great. Then on other days, I feel that it is a bit of low quality time spent out there for my students. I can then almost picture myself as one of my old drill sergeants in the army, shouting at the new recruits to get down and crawl in the boot camp mud. "Expand this function into a Taylor series now!" Making them do some things that everybody that has done science at the university has gone through since the late days of Taylor himself. It is neither that pleasant, nor interesting for them, but it has to be done

Torbjörn Lundh
Reader, Department of Mathematical Sciences, University of Gothenburg and Chalmers University of Technology

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nevertheless. Then, most of the other days, I have probably a more realistic view of the world and my teaching in general. It is an important and challenging job.

What is high quality teaching?

Let me now try to exemplify some of the complexities, or paradoxes, one can meet in a learning environment. Does creating a warm cozy atmosphere always give the best learning environment? We are all being raised to be polite and friendly and we naturally try to do the same in the classroom. We believe that it is vital that the classroom feels safe for the students in order for them to get the courage to ask any type of questions in order to travel on new paths. Having said that, let me here present two examples to indicate the complexity of the meaning of efficient teaching, where a not-to-recommend teaching styles made me, as a student, work harder, and learn more than I otherwise might had done.

The rude teacher

You might have experienced one or two of these teachers yourself? The obnoxious, aggressive type who likes to tease, and "put down" the students. There is one teacher in particular I have in mind when I consider my student years. The strange thing about it is that I learned a lot during his course. The main driving force for me, studying harder, was my attempts to be able to correct the (frequent) errors he made on the blackboard, as my little revenge for his rude attitude towards us. I still remember things he discussed with us. As a contrast, I can remember later being to some very fun, harmonic, and smooth lectures, but I cannot recall what they were all about. Compare this with the highly individual learning experiences in classrooms teaching described in (Nuthall, 2005).

The thought provoking teaching

As a student, you can sometimes meet a non-rude teacher that nevertheless makes you feel at unease whatever the reason might be. Maybe it is the "personal chemistry", the teaching style, or even the specific curriculum itself. An example from my own experience that comes to mind here is a pedagogical course I took as a graduate student. I felt the teacher put the focus on completely the wrong things, and on top of that, that those things were probably soon to be considered out of fashion. This made me discuss, read, and think a lot about the issues the course should have, in my opinion, been about. My friend Sakarias Åkerman, a high

school teacher, pointed out that even though I thought the course was not good, he had never seen me so engaged, and learning so much before. "This is probably the best course you have taken", he remarked. He was probably right. Today I would not have reacted as negatively, but on the other hand, I would not have gotten that engaged. I was frustrated that "practical knowledge" about teaching was a bit frowned upon, while so-called theories were put into focus. I can today clearer see the importance for any teacher to be aware of current theoretical studies about learning, but I still think it is a central problem that practical knowledge seems by some to have less "value" than theories.

The interplay between teaching and research

I believe that mathematics is somewhat special in that there are very limited channels of "usefulness" between the research and the teaching of calculus. Each time I do a Taylor series expansion I have to be fully focused, although I have done it hundreds of times before. And there are only slim chances that these concrete skills will ever be essential for future pure mathematical research. On the other hand, you cannot tell your students what you are really doing when you are not teaching or preparing the lectures. That, I think, is quite a serious problem. When you study mathematics at the university level, you should not only get the basic manipulative skills, but also a glimpse of a present research horizon. It is essential for the understanding of the subject that mathematics is far from "all said and done" but there is a lot of creation and exploration out there, and many new questions arises all the time. This is an insight I want to share with my students, even if it is impossible for them to understand the specific details at the time.

The interplay between pedagogical theory and teaching skills

The university teacher of today has to have more than one skill. The university teaching role is, in my opinion, in many ways getting closer and closer to the high-school teacher twenty-five years ago. There is even an informal well-known term for this process in Swedish – *gymnasifiering* ("high-school-ification"). She or he is also expected to be able to follow the present pedagogical research. After taking my own pivotal, but short, pedagogical training course (including the interview below), it was hard for me to see how a theoretical training in the pedagogical field would enable me to gain any "teaching skills" whatsoever. On the other hand, I could think of many examples where the lecturer's pure subject knowledge diffuses through the most hopelessly executed

teaching performance in such a way that it is almost impossible as a student not to be excited. See the "unaware" lecturer presented in figure 1 below or the PGK-model[1] presented by Lundh (1994) (see also Ma, 1999, p. 70). So what is the correlation between knowledge in pedagogical/didactic theory and teaching skills (whatever that might be)? I was convinced that this was an important question that would have been well studied, but surprisingly I could not find any single work thoroughly and soundly addressing this issue[2]. It is of course a very hard question to ask, but as all scientific branches have to clarify their potential usage and role in society and in the grand scheme of science (Wilson, 1998) the same should be true for the pedagogical sciences. How would one else be able to motivate prospective teachers to seriously study these fields, and agencies to fund research?

An interview with a celebrated teacher

Here follows an interview with the mathematical lecturer Anders Vretblad at Uppsala University October 20, 1994 translated from Swedish (Lundh, 1994). He is one of my absolute favorite mathematics lecturers, and one of the first teachers I met at the university. I must say that his answers came as a surprise to me at that time. Since then, I have met many other skillful lecturers with similar experiences and thoughts.

> *When did you get the Uppsala University Pedagogical Price?* The first time the prize was awarded, which was 1986.
>
> *What qualities do you have to get that reward?* I guess others should answer that. There was a speech by rektor (i.e. president) H:son Holmdahl, but I was too excited then to remember anything of that today. I think something was mentioned about my new book in introductory algebra, and that my lectures were not completely devoid of humorous remarks.
>
> *Do you think one can teach general pedagogy?* I do not know. I would never function outside my subject. Even though I am interested in history, I would never be able to teach that, simply because I would not have anything to say due to my lack of knowledge of the methodology and "style" in history teaching.
>
> *How can you become a better teacher?* You have to realize that the subject is hard (Anders Vretblad grabs two finals in calculus I. One made by his own hand, and the other by someone else). My students are working on this one at this very moment. But this one I have not

even tried to do myself, and I would not dream of showing it to my students. Look at this integral!

It seems to be more tedious than hard. That's right, very tedious. I bet that no one will get the right answer on that one, even if most of them know what to do. It is only a lot of boring calculations without finesse. And look at this last problem! It seems to be from a grad course in measure theory rather than this first course. Maybe it is not that hard, but I have not been able to force myself to digest the text of the problem. It cannot look like this! You have to be able to put yourself in the students' position!

Have you tried any modern form of teaching technique? There has been a lot of talk about PBI[3] *for example. Is that something you have tried?* Well, not really. A couple of years ago, we tried a form of tutoring at the civil engineering school here. The students were divided into groups of six or seven people to promote more discussion-based teaching. It didn't work at all. The engineering students never had anything to say. It became just like ordinary lecturing, with ordinary one-way communication for the mini-groups. It is possible that we the teachers were not competent enough to make it work. It is also possible that the subject itself, mathematics, is not that suitable for that kind of teaching. Maybe it would work on some special courses, like courses about differential equations. There, one could start from a real problem and build the theory from that point. Books have been written about such successful experiments.

Do you prepare a lot? Yes, preparations take time. But nowadays I mostly just put down keywords. In the beginning I wrote down whole sentences which I would say, and almost read word by word at the lectures. After a couple of years, I stopped preparing completely, since I noticed that I never followed my own notes anyway. But that didn't work for long, so I then turned into my present keyword technique.

What is your "drive" as a teacher? I want that others would discover what I discovered. I would like to share my "aha-moments".

What does the research mean for the teaching and vice versa? Well, since the frontier is so far away in our subject, it is very seldom one can pull up some new results in the teaching. It is even harder to imagine that one would get some fruitful research ideas in the teaching situation, but you can of course always "get an apple falling down on your head".

Wouldn't you have been an equally good teacher without your PhD degree? No, I would not think so. I would not have been able to write these lecture notes in Fourier analysis if I hadn't done work in that field. Fourier analysis is my subject.

How is your present research? It is close to nothing. I took my grad courses without any problem, but when I started to write my thesis, a tiredness came over me. Anyway, I got it together, after some delay. But I felt that I was more suited to teach mathematics then to create own, new mathematics.

Is the teacher a theater man? Yes, at least at the lectures. You take a deep breath and you come up on the other side of the pool after 45 minutes. Sometimes you can be so absorbed in your teaching that you forget about the world outside the lecture hall.

That sounds like any old artist. Yes, sometime it feels like singing with OD[4].

But the difference is that you are completely alone as a teacher? That's right. It may sound a little harsh, but I believe that one has to have some talents for acting to be comfortable as a teacher.

But are you really suppose to act? Shouldn't a good teacher be natural? In the lecture hall, you are supposed to promote a dialogue, and to do that you have to be natural; if worse comes to worse, you have to work to be natural. Look at Winston Churchill for example. He was a brilliant speaker, but not at all a natural speaker.

What does your own formal pedagogical education look like? As a grad student, I went on a two days education in "overhead technique", a technical tool that was very "hot" then in the middle of the '60s; voice technique, which was very good; and a dose of pedagogy in the form of a lecture by pedagogy professor W. Sjöstrand. It was a remarkable lecture. The subject was the importance of writing down keywords on the black board. His lecture was very monotonic and slow, many fell asleep. He finished without having written down any keyword on the board or anywhere else.

My second and last contact with pedagogues was 1974 and was very unfortunate. I just had become director of undergraduate studies and was supposed to get some more education in pedagogy. The university sent me away on a boarding school for two weeks. The leaders, if you could even use such a word, made us do psychological exercises, so called sensibility exercises, which could for example be

> "stare into each other's eyes as long as you could take it". The course had a deep impact on me. I even resigned as the director after that. I decided never to have anything to do with pedagogues again.
>
> *How would you like to see the teacher training?* As a matter of fact, I have just started to think about that, since we are going to start tutoring teaching grad students. I will put the emphasis on auscultation. They will listen to me and I will listen to them. It will be a kind of "learning by doing". I do not think one has to make it so complicated. It is enough with common sense, structure and time for preparation.

Needless to say, this interview was a turning point for me when it came to evaluating pedagogical knowledge. Before this meeting, I had the idea that to improve as a teacher, it was essential to study pedagogy – now what to believe and how to improve?

Compare the last paragraph in the interview above with a citation of Dubisch (1963) in Steuben and Sandford (1998, p. 4):

> Much, much more has been written about good teaching, but all the advice I have seen can, I think, be boiled down to about this: The good teacher is a human and mature person who knows his subject thoroughly, has a keen interest in it, and tries to get it across to his students in a thought-provoking fashion.

At my post-doc at State University of New York (SUNY), I heard in the common room a different variant of this viewpoint: "Teaching skill is what your mother taught you". That short sentence somehow demystifies the amount of theoretical pedagogical knowledge needed of a good teacher. On the other hand, it puts the focus on the so-called empathic qualities and by that promotes the somewhat fatalistic feeling that it is hard to see how one can improve and thus grow as a teacher. Let me now discuss a few books that I found useful while pondering that question how to improve.

A few interesting books

Twenty years before the blackboard is a concise, funny and enlightening book (Steuben & Sandford, 1998). It consists of two parts. The first is a very nice description of the main author's way into profound math teaching filled with examples and quotations. The second part of the book is based on Steuben's mathematical scrapbook, which consists of selected humorous stories and tricks.

Steven Krantz at Washington University in St. Louis has written *How to teach mathematics* (Krantz, 1999) which I find very useful for a university teacher. There are many reasons why one should read this book. It is very down to earth with a lot of concrete advice you can take advantage of right away. He even has a section about how to dress. That is very far from most text in mathematical didactics today. Krantz is a very efficient and funny writer who is not only an excellent researcher, but also superb teacher. He has taken the effort to describe situations and thoughts he has collected during the years. He has also been in charge of the internal mathematical teachers training in St. Louis.

The courage to teach (Palmer, 1998) is a quite different book. It is more thought provoking, or maybe even poetic, than down to basics. According to the author his book is aimed at a general group of teachers: "This book is for teachers who have good days and bad, and whose bad days bring the suffering that comes only from something one loves. It is for teachers who refuse to harden their hearts because they love learners, learning, and the teaching life" (ibid., p. 10). He then goes on to describe the underlying principles put forward in the book: "This book builds on a simple premise: good teaching cannot be reduced to technique; good teaching comes from the identity and integrity of the teacher" (ibid., p. 10).

Palmer discusses the importance of the underlying subject in the learning process. "Perhaps the classroom should be neither teacher-centered nor student-centered but subject-centered" (ibid., p. 116). Compare this viewpoint with Steuben's.

> The scholar or ambitious student is the opposite of most classroom students [...] Scholars thrive in a competitive environment. They respect intellectual power and have contempt for stupidity. Guess who will become the teachers for the next generation? The scholars of the last generation [5]. The problem is that the scholars' attention to book knowledge can too easily become a hazard to effective teaching. Why? Because the scholar is in love with his subject, and the subject is secondary to a student's classroom experience.
>
> (Steuben & Sandford, 1998, p. 15)

These two opposite viewpoints are a good example both of the paradoxes in teaching mentioned above, and that mathematics is perhaps an extreme subject in this sense. Chapter VI deals with the meager collegial situation, "Though we teach in front of students, we almost always teach solo, out of collegial sight" (Palmer, 1998, p. 143).

Palmer also addresses the teacher evaluation system, where he is highly critical to rely on "the artifacts" of the students' survey. [6]

> There is only one honest way to evaluate the many varieties of good teaching with the subtlety required: it is called being there. We must observe each other teach, at least occasionally – and we must spend more time talking to each other about teaching. Then, when the time comes for promotion and tenure decisions, we will have real information to work with, rather than the statistical fictions with which we now manipulate decisions. (ibid., p. 143)

I have myself had senior lecturers making unannounced visits to my lectures both at KTH and at SUNY where I also took the other part, when visiting my teaching assistant's lessons. These things, being initially uncomfortable, felt rather constructive. I got and gave vague but still constructive criticism. I guess that much of my positive feeling about this is the thought of the alternative: that no one except my students would care about my work at all. In this sense I strongly agree with Palmer when he talks about "being there". A slightly different approach one can find in China (Ma, 1999, p. 136), where "teaching research groups" are an important tool for teachers with collegial discussions and promoting life long learning.

As a summary of the books might be that both Stueben's and Krantz's books are both highly inspiring and directly useful for a math teacher, while Palmer is also inspiring, but more on a "deeper" level. I would say that both kinds of books are needed for a developing teacher no matter on what stage. All of the books above give also a strong emphasis on the practical side of teaching, teaching as a skill to develop and to practice. The scientific grounds of pedagogy are not the focus, but teaching more as an art.

The Jyväskylä-categorization

In order to clarify the different approaches you might take as a lecturer, I suggested in (Lundh, 1994) a categorization of mathematical lecturers into four basic groups:

- *The complete*. This teacher never has enough time and wants the clock to slow down so he would manage to present the complete picture with all its important details. He works intimately with the blackboard and aims at presenting an alternative book on the board.

- *The defensive*. There is usually more time than material left at the end of the lecture. To be honest, he is not so comfortable, neither in front of the students nor with the specific topic of the day. He

is usually not foreign to modern pedagogical methods where the student themselves should take active part in the lectures.

– *The sparkling.* This person is quite comfortable on a stage. Probably on any stage, but preferable at a lecture where he or she can run his well-rehearsed show on a subject he masters. Often this lecturer is a musician or actor on weekends.

– *The unaware.* This lecturer is truly passionate about his or her research. She is so comfortable in the often world-leading position, that she has nothing more to prove for the audience. Hence she can easily reveal her ignorance, and above all, does not have to "show-off" with complicated arguments, but can on the other hand take some liberties to explain things in a more intuitive manner.

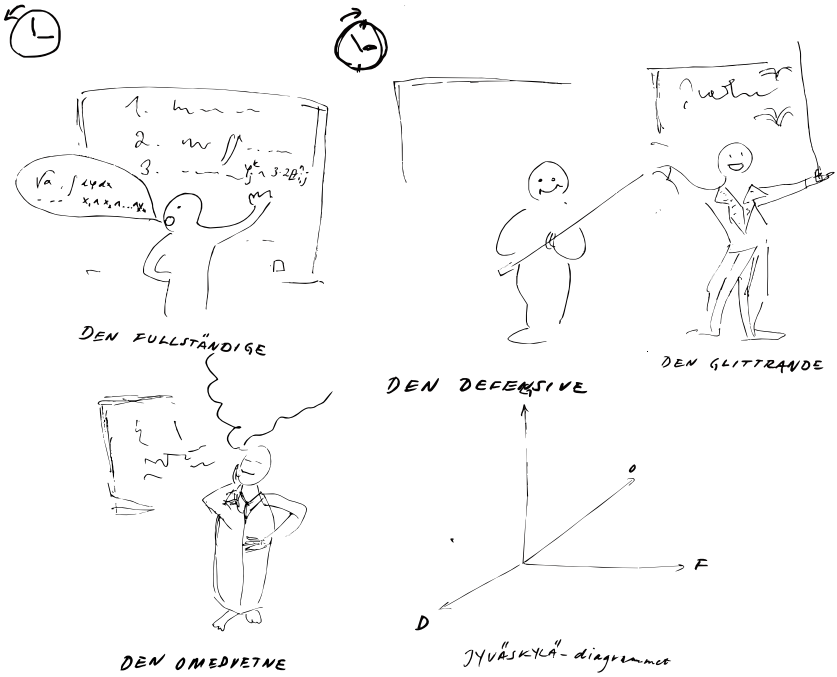

Figure 1. *The Jyväskylä categorization. From upper left: The complete lecturer, the defensive lecturer, the sparkling showman, and the unaware guru. The last figure is a depiction of the 4-dimensional Jyväskylä diagram where one could localize a specific lecturer. The sketches are taken from (Lundh, 1994).*

If I should try to mark my own position in the Jyväskylä diagram given as the last picture in figure 1, I believe that my style has changed from the

time I wrote (Lundh, 1994) until today. When I was a graduate student, I probably was somewhere close to the plane spanned by the defensive (due to my inexperience and thesis focus) and the sparkling (due to my music interest), as indicated in the diagram. Today, I can clearly see all four ingredients in my own teaching, but as always, it is much harder to classify oneself than some other. My role model from the interview above, I would definitely place plane the plane spanned by the sparkling and the unaware.

Pedagogy and Politics

Nuthall (1998) presents a personal account in one of the best texts about teaching and learning I have came across. He describes his almost life-long quest to understand classroom learning as a long journey, with hills and valleys and many detours as well, in a manner that you realize that this man really wants to know what classroom teaching and learning is all about, even if it means to find unexpected and contradictory results. He argues that quite a bit of what goes on in a classroom are based on myths and rituals. Nuthall also points out a future direction of the field where deeper knowledge of individual learning will be needed. I also hope that future directions of the research about teaching and learning will be multidisciplinary. It has to be since the questions are so complex and far-reaching. Furthermore, it is absolutely worth perusing since it deals with our children and their future. See for example (Bartolini Bussi & Bazzini, 2003) who argue for an improved dialogue between different fields concerning these questions.

Another piece of work that is relevant to mention here is Gustavsson and Myrberg (2002) which is a report (in Swedish) from Skolverket (National Agency for Education) which has a chapter devoted to teacher competence and how it is best improved. Here best should probably be understood as "most cost-effective". The research overview is to a great extent based on (Darling-Hammond, 1999). Some of the quoted results was quite contradictory to my own personal experience, such as that the pedagogical training was more important than the subject knowledge, in such a way that it had a higher degree of explanation of student results! (see Gustavsson & Myrberg, 2002, p.125). Could that be true? That would certainly go against my own viewpoint and much of the things I have referred to above including the investigation in (Ma, 1999) where the key-concept PUFM (profound understanding of fundamental mathematics) is identified and discussed; and where the question "Can pedagogical knowledge make up for ignorance of the concept?" is raised and answered (pp.70–71)[7]. It got even more puzzling, since in the main

source that Gustavsson and Myrberg (pp. 127–128) refer to, a statistical investigation was described. One of the surprising results presented was that although the successful teacher needs subject knowledge in his/her field, this knowledge becomes irrelevant or less meaningful at a certain level. I did find that statement so strange and curious so I had a look at the source presented in table 2 in (Darling-Hammond, 1999). There one can see that the strongest incitement, in six out of six cases, for good student achievement (NAEP-test) was a teacher with full certification and a major in the field. However, as was pointed out in (Walsh, 2001), one could also learn that there was only one case showing significant positive correlation if the teacher was only fully certified. That is, the pure subject knowledge made a highly significant contribution. For some reason Darling-Hammond did not acknowledge this straightforward fact from her own table. Hence, her conclusion seems therefore to be biased against the importance of subject knowledge. Unfortunately, her conclusion gets spread around anyway as a well accepted result as for example by Gustavsson and Myrberg (2002).

Just to indicate how hotly and passionately this issue is debated, let me mention that Walsh's highly critical paper got a long response by Darling-Hammond (2002), which in turn got criticized by Walsh and Podgursky (2001), where the issue how to read table 2 was still under debate. Furthermore, the other surprising quote above about the limited importance of subject knowledge (originally based on Ferguson & Womack, 1993) was also being under debate. Podgursky states (Walsh & Podgursky, 2001, pp. 12–13) that the statistical method used there was simply incorrect. I have to agree, the method was misused in a too common manner (described for example in Rencher & Pun, 1980).

As far as I can see, following the criticism and contra-criticisms; and going to some of the original sources, I would not recommend to refer to Darling-Hammond's work without checking it carefully first. It is therefore quite serious when such a source is used as central argumentation concerning educational strategies in an official report from the Swedish National Agency for Education. Having come so far in my digging, I realized that the questions under debate long ago had left science [8] and gone deep into politics [9]. Another criterion that is a great divider between scientists and politicians is the ability to change. In politics, it is a sign of weakness and unsteadiness if one changes directions or ideology, where in science, such changes are of course painful but are considered as a virtue. Compare Nuthall's approach where he describes many turning points and changes of views, with the steadfastness of Linda Darling-Hammond who refuses to change opinion even when faced with concrete facts in her own data material.

Conclusion and Discussion

I think calculus is a wonderful intellectual achievement of the collective human mind. Even without its direct usefulness, it would be worth learning [10]. Although I have taught calculus for some time now, I still feel I just started and I have good hopes that I will find more personal angles, and approaches, when trying to create efficient learning situations in the future. Maybe even something along the developmental process described in Steuben and Sandford (1998)? This essay has been a vehicle for me while searching for such a path and I hope this text might be somewhat interesting, thought provoking, or maybe even useful, for others interested in teaching mathematics.

As a spinoff of this process, I have stumbled upon two great challenges currently faced by didactics as a field. Since it deals with very complicated and complex dynamics and processes occurring for example in a classroom (see Nuthall, 2005), the field cannot be expected yet to be as mature as other fields which have had a longer history but deal with processes and observations less elusive to catch (Wilson, 1998). Hence, didactics as a field has yet to prove that it is a solid science. The key to do so seems to lie in the ability to produce reproducible results in the spirit of Popper (Nuthall, 2005; see also Millar, 2007, 2008). At the same time the pedagogical research has yet to convincingly show it can produce results that are actually useful for improving teaching even for a skeptical teacher, see also Hargreaves (1996) and Carr (2006). This last point is naturally of even greater concern for the working teacher. One could argue that these questions should go together, but that is not always the case, since in order to get reproducible results, one has to look at questions less challenging or informative to the practically oriented teacher looking for advice. For example, a study was quoted which revealed that the correctness in the French teacher's pronunciation had correlation with the results their pupils obtained on pronunciation and listening tests (Gustavsson & Myrberg, 2002, p. 130).

Millar (2008) asks if we should look at the medical way to produce reproducible research. Before modern medicine research took its current form, new medical knowledge was not accumulated, but rather contradicting older "truths". We see a similar situation when it comes to research in pedagogy/didactics as was pointed out by Hargreaves (1996, p. 2).

In the future, I would like to see both sides of the field, the scientific part and the practical part, being explored together in harmony. Not only with each other but also in a context of other related fields in a much more close collaboration than we see today, maybe something along the line described in Wilson (1998), and Bartolini Bussi and Bazzini (2003) where interesting abstract results would not be treated with greater respect than practically useful advice, or even skillful plain teaching.

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Notes

1 Pedagogy Generated by Knowledge

2 I found a couple that claimed to have such intentions, such as studies by Linda Darling-Hammond at Stanford, see Darling-Hammond (1999, 2002).

3 At the time of the interview, I naively used the acronym PBI, but my ignorance was reviled in a later discussion where I was informed that PBI was old news; the new thing was called PBL.

4 Orphei Drängar, a world touring male choir from Uppsala

5 I am afraid I don't believe that this is absolutely true anymore, at least not for budding school teachers of mathematics here in Sweden. Personally, I strongly believe the math teachers should be paid much better, and be given better working conditions. That would increase the attractiveness of the occupation back to the higher levels where it belongs.

6 It could in fact be worse. In some cases in Sweden, teacher evaluations are based on only the applicant's own written statements of the mandatory "teaching philosophy" in the applications.

7 If you only have time to read a part in (Ma, 1999), let it be Chapter 4 where Ma presents a beautiful question to teachers how they would react if an excited student presented an own discovery which said that if you increase the perimeter of a closed figure, the area also increases. This chapter shows that profound knowledge is essential for a teacher, including working skills in mathematical methodologies. Knowing how to search for information in the literature is not enough, which is clearly illustrated here since the above "discovery" was not correct.

8 If it ever had been there in the first place.

9 Following the press covering, Darling-Hammond was close to become the Education Secretary under president Obama. In the end, Arne Duncan was chosen.

10 Having said that let me nevertheless recommend an insightful text about the development of the Swedish school curriculum in mathematics (IKUM, 2008).