A Time-indexed Formulation of a Flexible Job Shop Problem with a Limited Number of Fixtures

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1 Introduction

The *multitask cell* (MTC) at Volvo Aero Corporation is a flexible job shop containing ten resources aimed at being flexible with regard to both product mix and processing types, increasing the degree of machine utilization, and reducing product lead times, compared with the ordinary job shops at the production site. The MTC is intended to carry out a large variety of jobs, since five of the cell's resources are multi-purpose machines that are able to process three different types of operations (turning, milling, and drilling). Each part to be processed in the MTC follows a specific routing through the set of resources, consisting of three to five so-called *route operations*, starting and ending by the mounting and removing of fixtures at one of the three set-up stations. There are storage areas inside the cell for the storage of parts before and between processing.

Each part typically visits the MTC multiple times on its way to completion. One such visit to the MTC is called a *job*. Each job occupies a fixture, specially manufactured for each specific job type. The number of fixtures of each type is limited. As each job can be processed only in a subset of the multitask machines, the scheduling of the MTC is a complex combinatorial problem, a so-called *flexible job shop problem.*

2 Related Work

The flexible job shop problem (FJSP) is an extension of the job shop problem, in the sense that each operation may be scheduled in more than one of the machines; see Baykasoglu and Özbakir (2012). Time-indexed formulations using decision variables which equal 1 if the corresponding job starts at a specific discrete time step, and 0 otherwise, are found in Sousa and Wolsey (1992) for non-preemptive single machine scheduling problems, and more generally in Wolsey (1997) for production planning and scheduling problems. The formulations using variables for each discrete time period lead to very large models in terms of numbers of both constraints and variables, but typically yield better lower bounds than other mixed integer linear programming (MILP) formulations of scheduling problems; see van den Akker *et al.* (2000).

The operations research literature comprises however very few time-indexed formulations of the job shop problem, we have found one (Azem *et al.* 2007) and none of the flexible job shop problem. The most common way of formulating the FJSP is to use variables introduced by Manne (1960) for the jobs' starting times along with decision variables for the ordering of jobs, defined as y_{jq} equals 1 if job j precedes job q, and 0 otherwise. Some examples of so-called *Manne family* models for the job shop problem and the FJSP can be found in Özgüven *et al.* (2010), Fattahi *et al.* (2007), and Azem *et al.* (2007). In

Thörnblad *et al.* (2010) we present a Manne family model of the problem of scheduling the multitask cell.

3 The machining problem with fixture constraints

We consider here a subproblem of the problem of scheduling all ten resources of the multitask cell, namely the problem of finding an optimal sequence of operations for each of the five multi-purpose machines. The optimal solution of this so-called *machining problem* can then be used as input data to a model which generates a feasible schedule for all ten resources; see Thörnblad (2011).

3.1 Problem definition

The machining problem can be defined as a set of n jobs, $\mathcal{J} = 1, \ldots, n$ to be processed on the set $\tilde{\mathcal{K}}$ of multipurpose machines during $\tilde{p}_j$ time units. Each machine can only process one job at a time and each job is allowed to be processed only in a subset of the multitask machines; a parameter λ_{jk} is valued 1 if job j can be processed in resource k, 0 otherwise. Some of the jobs are subject to precedence constraints, since they are to be processed on the same physical part; hence, for some jobs the corresponding part is inside or on its way to the MTC for the processing of a preceding job in the routing, before making another round in the factory and, finally, reaching the MTC for the processing of the job in question. The pairs (j, q) of all such jobs adjacent in the routing form the product set $\mathcal{Q} \subset \mathcal{J} \times \mathcal{J}$. For each $(j, q) \in \mathcal{Q}$, the planned lead time between the completion of job j and the start of job q, is denoted by $\tilde{v}^{\mathtt{m}}_{jq}$.

The planning horizon of the schedule is divided into $T + 1$ intervals, each of length ℓ hours. The index $u \in \mathcal{T} = \{0, 1, \ldots, T\}$ denotes the interval starting times. Since the resources are often occupied by the processing of previous jobs at time 0, the parameter $\tilde{a_k}$ is introduced, denoting the first time resource k is available. Each job has been assigned a release date, $\tilde{r}^{\mathtt{m}}_j$, denoting the time when the job is available for processing, and a due date $\tilde{d}_j$. The parameters $\tilde{p}^{\mathtt{am}}_j$ and $\tilde{p}^{\mathtt{pm}}_j$ denote the sum of the processing times of all operations preceding and succeeding the machining operation of the job, respectively.

Each job occupies a fixture during the whole visit in the MTC. Each fixture type $f \in \mathcal{F}$ is specially designed and can only be used for a subset $\mathcal{S}_f$ of jobs. Since they are very expensive, only α_f fixtures of each type are available.

The value of the parameter T has to be large enough such that the time horizon $[0, (T+1)\ell]$ contains an optimal schedule. A small value of T is, however desirable, since this means that the computation times become shorter. This is due to the fact that the number of variables and constraints in the time-indexed formulation is a multiple the number of time intervals. We determine a suitable value of T using a heuristic; see Thörnblad (2011).

3.2 A time-indexed formulation

The machining problem with fixture constraints can be formulated as that to

$$\text{minimize} \qquad \sum_{j \in \mathcal{J}} (a_j s_j + b_j h_j), \tag{1a}$$

$$\text{subject to} \qquad \sum_{k \in \tilde{\mathcal{K}}} \sum_{u \in \mathcal{T}} x_{jku} = 1, \qquad j \in \mathcal{J}, \tag{1b}$$

$$\sum_{u \in \mathcal{T}} x_{jku} \leq \lambda_{jk}, \qquad j \in \mathcal{J}, k \in \tilde{\mathcal{K}}, \tag{1c}$$

$$\sum_{j\in\mathcal{J}}\sum_{\nu=(u-\tilde{p}_j+1)_+}^{u} x_{jk\nu} \leq 1, \qquad k\in\tilde{\mathcal{K}}, u\in\mathcal{T} \tag{1d}$$

$$\sum_{k\in\tilde{\mathcal{K}}}\left(\sum_{\mu=0}^{u} x_{jk\mu} - \sum_{\nu=0}^{u+\tilde{v}_{jq}^{\mathtt{pm}}} x_{qk\nu}\right) \geq 0, \qquad (j,q)\in\mathcal{Q}, u=0,\ldots,T-\tilde{v}_{jq}^{\mathtt{pm}}, \tag{1e}$$

$$x_{jku} = 0, \qquad (j,q)\in\mathcal{Q},\ k\in\tilde{\mathcal{K}},\ u=T-\tilde{v}_{jq}^{\mathtt{pm}},\ldots,T, \tag{1f}$$

$$\sum_{j\in\mathcal{S}_f}\sum_{k\in\tilde{\mathcal{K}}}\sum_{\nu=(u-(\tilde{p}_j+\tilde{p}_j^{\mathtt{pm}})+1)_+}^{\min\{u+\tilde{p}_j^{\mathtt{am}},T\}} x_{jk\nu} \leq \alpha_f, \qquad f\in\mathcal{F},\ u\in\mathcal{T}, \tag{1g}$$

$$\sum_{k\in\tilde{\mathcal{K}}}\sum_{u\in\mathcal{T}} u x_{jku} + \tilde{p}_j^{\mathtt{pm}} = s_j, \qquad j\in\mathcal{J}, \tag{1h}$$

$$h_j \geq (s_j-\tilde{d}_j)_+,\ j\in\mathcal{J}, \tag{1i}$$

$$x_{jku} = 0, \qquad j\in\mathcal{J},\ k\in\tilde{\mathcal{K}},\ u=0,\ldots,\max\{\tilde{r}_j^{\mathtt{m}};\tilde{a}_k\}, \tag{1j}$$

$$x_{jku} \in \{0,1\}, \qquad j\in\mathcal{J},\ k\in\tilde{\mathcal{K}},\ u\in\mathcal{T}, \tag{1k}$$

where $(u)_+ := \max\{0;u\}$. The objective (1a) is to minimize the weighted sum of the variables completion times s_j, and tardiness h_j, with weights a_j and b_j, respectively. The main variable in the formulation is the variable x_{jku}, which is 1 if job j starts in resource k at time step u, 0 otherwise. The constraints (1b) and (1c) ensure that every job j is processed exactly once and scheduled in an allowed resource, respectively. The constraints (1d) regulate that only one job at a time is scheduled in a resource k. The constraints (1e)–(1f) ensure that job j is scheduled to start at least the planned lead time $\tilde{v}_{jq}^{\mathtt{pm}}$ before the start of job q, for all pairs $(j,q)\in\mathcal{Q}$. The capacity constraints on the number of fixtures occupied at each time interval are formulated in (1g). The constraints (1h)–(1i) define the completion times, s_j and the tardiness, h_j. The constraints (1j) make sure that job j is scheduled after its release date in an available resource.

A check to ensure that the chosen value of T is large enough and that the optimal schedule found is indeed optimal for all larger values of T, it suffices to check whether if $T \geq \max_{j\in\mathcal{J}}\{s_j\} + \max_{j\in\mathcal{J}}\{\tilde{p}_j\}$ holds. If this is the case, there is enough room at the end of the schedule for any job to be scheduled in any other resource, and hence a larger value of T would not change the optimal objective value.

4 Computational results

Results from the model (1) with and without the fixture constraints (1g) have been compared with a Manne family model, the so-called *engineer's model* (model (7) in Thörnblad (2011)), for six real scenarios collected from the MTC during the autumn of 2010, from which instances were created with $5 \leq n \leq 60$. A limit of 10 000 seconds was set for the computation time (clocktime) and the computations were carried out using AMPL-CPLEX12 on a computer with two 2.66GHz Intel Xeon 5650, each with six cores (24 threads), and its total memory was 48Gbyte RAM. In Fig. 4 the mean CPU-times are plotted for the engineer's model without fixture constraints (Thörnblad 2011), and the model (1) with and without the constraints (1g) included. The objective weights chosen were $a_j = 1$ and $b_j = 10, j\in\mathcal{J}$, so that the tardiness become the most important objective.

The model (1) outperforms by far the engineer's model, which could only solve small instances with $n \leq 15$. The time required to solve the model (1) was 6.5 minutes (clocktime) on average for scenarios with 45 jobs, which we have estimated to be the largest size of a realistic instance for the scheduling of the coming shift; the longest computing

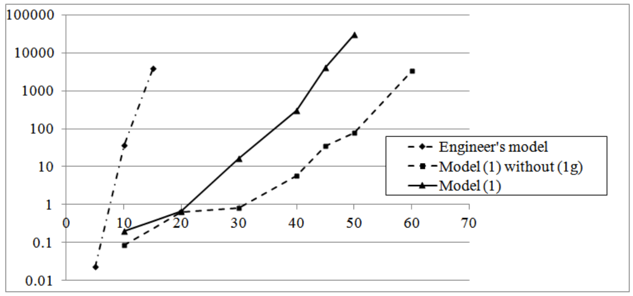

Fig. 1. Mean CPU-times for the engineer's model without fixture constraints, and for the model (1), as is and with the constraints (1g) removed.

time required was 21.5 minutes (clocktime). When the constraints (1g) were removed, the average computation time for model (1) was 15 seconds (the longest was 60 seconds).

5 Conclusions

We have formulated a time-indexed model, which outperforms by far the engineer's model from Thörnblad (2011), which is a model of the Manne family, with respect to computation times and the sizes of instances that they are able to solve using standard optimization software. The adding of the fixture availability constraints resulted in an increase of the computation time by a factor of 23 on average for the time-indexed model. However, these computation times are still acceptable for finding a schedule for the coming shift in a real application.

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