Cooling of a Suspended Nanowire by an ac Josephson Current Flow

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We consider a nanoelectromechanical Josephson junction, where a suspended nanowire serves as a superconducting weak link, and show that an applied dc bias voltage can result in suppression of the flexural vibrations of the wire. This cooling effect is achieved through the transfer of vibronic energy quanta first to voltage-driven Andreev states and then to extended quasiparticle electronic states. Our analysis, which is performed for a nanowire in the form of a metallic carbon nanotube and in the framework of the density matrix formalism, shows that such self-cooling is possible down to the ground state of the flexural vibration mode of the nanowire.

Nanoelectromechanical systems (NEMS) have, over the past two decades, been a very active field of both fundamental and applied research. With typical dimensions on the nanometer scale, NEMS combine electronic and mechanical degrees of freedom for novel applications. These include small mechanical resonators for ultrasensitive mass spectroscopy [1,2] and position sensing with close to quantum limited resolution [3,4]. The prospect of a controlled fabrication of NEMS devices whose physics is ultimately governed by the laws of quantum mechanics is a major driving force in the field today.

Typically, nanoelectromechanical systems comprise a mechanical resonator coupled to an electronic system used for both actuation and detection. Because of the high mechanical vibration frequencies and the exceptionally high quality factors recently achieved, these systems allow for very low energy dissipation and extreme sensitivity to external stimuli. To make full use of their potential, much research has recently focused on the possibility to cool the mechanical subsystem in NEMS to its vibrational ground state. Several theoretical papers have invoked sideband cooling or dynamical backaction in order to predict electromechanically induced cooling of mechanical resonators to the ground state [5–8]. The best experimental result so far, corresponding to a vibron occupation number of 3.8, is due to Rocheleau *et al.* [9]. They used an external electromagnetic field as an energy transducer for sideband cooling of the mechanical resonator. Similar results have been achieved by cooling with dynamical backaction [10].

Here, we propose a new cooling method, which utilizes the unique electronic properties of a Josephson junction. We consider an *S-N-S* junction which is composed of a nanowire in the form of a metallic carbon nanotube suspended between two superconducting leads as in Fig. 1. Below we demonstrate that such a nanomechanical weak link has the capacity to self-cool, if biased by a dc voltage V, through the transfer of energy from the flexural vibrations of the nanowire to a bath of electronic quasiparticle excitations. Recently, it has been experimentally shown that for systems similar to the one considered here it is possible to cool nanomechanical objects below the ambient temperature through the transfer of mechanical energy to electronic quasiparticle states [11,12]. Similar to these works, the cooling process to be proposed is an inherent property of the system, which makes the suggested mechanism a promising candidate for truly quantum mechanical manipulation of mechanical resonators.

The electromechanical coupling required for our cooling mechanism can be achieved by applying an external magnetic field perpendicular to the length of the nanowire. Displacement of the nanowire then couples the mechanical and electronic degrees of freedom through the Lorentz force. For an *S-N-S* junction that is short compared to the coherence length in the superconductors, the ac Josephson dynamics induced by the weak dc bias voltage
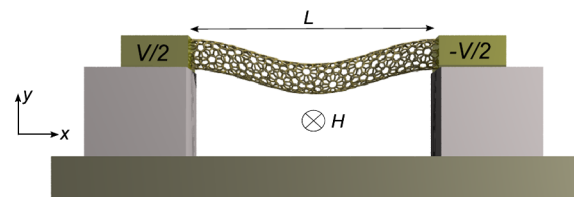

FIG. 1 (color online). Schematic diagram of the Josephson junction discussed in the text. A suspended carbon nanotube of length L forms a weak link between two superconducting leads. An external magnetic field H gives rise to a Lorentz force that couples flexural vibrations of the nanowire to the electrical current that is carried by Andreev states through the junction if it is biased by a voltage V. Cooling is achieved by the absorption of vibration-energy quanta as the electronic subsystem is excited across a gap in the Andreev spectrum (see Fig. 2).

can be expressed in terms of a pair of Andreev states, which is periodically created and destroyed in the junction. Between creation and destruction the Andreev levels evolve adiabatically within the energy gap of the superconductors; the level spacing first shrinks, reaches a minimum, and is then restored to its initial value as the Andreev states are dissolved in the continuum quasiparticle spectrum (see Fig. 2). Being thermally populated when created, the population of the Andreev levels become "overcooled" as they evolve towards the middle of the gap. Since the Andreev states are coupled to the nanowire vibrations, they can absorb a quantum of vibration and approach an equilibrium distribution through an electronic excitation from the lower to the upper Andreev level. The absorbed energy is transferred to the electronic quasiparticle continuum (and therefore transported away from the junction) when the Andreev states are dissolved. Cooling of the nanowire proceeds by repeated absorptions of vibrational quanta in subsequent cycles of the temporal evolution of the Andreev levels.

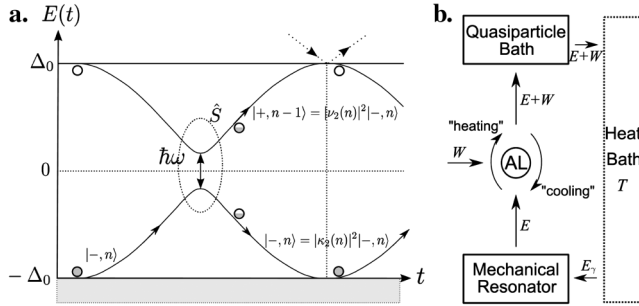To discuss the cooling mechanism quantitatively we consider the model Hamiltonian,

FIG. 2.   (a) Temporal evolution of voltage-driven Andreev levels (full lines) in the Josephson junction of Fig. 1. Here, $|\pm, n\rangle$ are states of the junction, $|\pm\rangle$ denotes the upper and lower Andreev level, and $|n\rangle$ is the quantum state of the nanowire. Transitions between $|-, n\rangle$ and $|+, n - 1\rangle$, through the emission or absorption of one vibrational quantum, can be modeled by a scattering matrix $\hat{S}$. The transition probabilities depend on the state of the nanowire through $|\nu_2(n)|^2 = 1 - |\kappa_2(n)|^2$. After one period, T_V, the partially filled Andreev levels join the continuum, creating quasiparticle excitations (represented by dashed arrows) and the electronic states are reset (filled and empty circles). Cooling of the nanowire occurs because absorption of vibrational quanta is more probable than emission at the ambient temperature. (b) Corresponding energy flow diagram. In each cycle the voltage-driven Andreev levels (AL) move some energy E from the mechanical resonator to a quasiparticle bath in thermal contact with a heat bath at temperature T. In this process the battery supplies an energy W. Only if the electronic and mechanical subsystems are coupled can E and W be nonzero. The resonator is coupled by dissipation to the same heat bath from which it receives an energy E_γ ($=E$ in the steady state). "Cooling" ("heating") refers to decreasing (increasing) the energy separation between the Andreev levels.

$$\hat{H}(t) = \mathcal{E}(\phi(t))\hat{\sigma}_z + \Delta_0\sqrt{R}\sin[\phi(t)/2]\hat{\sigma}_x + \hbar\omega\hat{b}^\dagger\hat{b}$$
$$+ \frac{2e}{\hbar}\frac{\partial\mathcal{E}(\phi(t))}{\partial\phi}LHy_0(\hat{b} + \hat{b}^\dagger)\hat{\sigma}_z. \quad (1)$$

Here, the first two terms describe the adiabatic dynamics ($\hbar\dot{\phi} \ll \Delta_0$) of the two level system formed by the Andreev states that carry the Josephson supercurrent through the (normal) nanowire [13,14]; $\hat{\sigma}_\alpha$ are Pauli matrices, Δ_0 is the order parameter in the superconducting leads, R is the normal state electronic reflection probability, and $\mathcal{E}(\phi(t)) = \Delta_0\cos[\phi(t)/2]$ is the energy of the Andreev states for the completely transparent junction with $\phi(t)$ the phase difference between the two superconductors. The latter depends on the bias voltage according to the Josephson relation $\dot{\phi} = 2eV/\hbar$. The third term in the Hamiltonian models the vibrating nanowire as a harmonic oscillator, taking only the fundamental bending mode into account; the operator $\hat{b}^\dagger$ ($\hat{b}$) creates (annihilates) one quantum $\hbar\omega$ of vibrational energy. The last term in (1) describes the electromechanical coupling of the current-carrying Andreev states to the motion of the nanowire through the Lorentz force. Here, e is the electronic charge, L is the length of the wire, y_0 is the zero-point oscillation amplitude, H is the magnetic field, and $(2e/\hbar)(\partial\mathcal{E}/\partial\phi)\hat{\sigma}_z$ is the current operator in the nanowire [15].

In the adiabatic regime, i.e., when $eV \leq eV_c = 4R\Delta_0$, it is convenient to use a basis set formed by the states $\psi_\pm(\phi(t))$ with corresponding energies $E_\pm(t) = \pm E(t)$, where $E(t) = \Delta_0\{1 - D\sin^2[\phi(t)/2]\}^{1/2}$ and $D = 1 - R$ [16]. In this basis, the Hamiltonian (1) reads,

$$\hat{H}_{\text{eff}}(t) = E(t)\hat{\tau}_z + \hbar\omega\hat{b}^\dagger\hat{b} + \Delta_0\Phi(\hat{b}^\dagger + \hat{b})\hat{\tau}_x. \quad (2)$$

Here, the Pauli matrices $\hat{\tau}_i$ span the space formed by the states $\psi_\pm(\phi(t))$ (see Fig. 2). In the last term of (2), $\Phi = 2LH\pi y_0/\Phi_0$ is the dimensionless magnetic flux threading the area swept by the vibrating nanowire; $\Phi_0 = h/2e$ is the magnetic flux quantum.

One notes that when $\phi = \pi$, the Andreev states $\psi_\pm(\phi = \pi)$, with energies $E_\pm(\phi = \pi) = \pm\Delta_0 R^{1/2}$, are symmetric and antisymmetric superpositions of states carrying current in opposite directions. Transitions between the Andreev states—induced by the nanowire as it vibrates in a transverse magnetic field—are therefore by far most probable when $\phi(t = t_0) = \pi$. Below we will therefore consider the resonant situation when $\hbar\omega = 2\Delta_0 R^{1/2}$ [17], which is the optimum condition for the proposed cooling mechanism. To proceed further we apply the rotating wave approximation, after which the Hamiltonian reads,

$$\hat{\mathcal{H}}_{\text{eff}}(t) = \begin{pmatrix} E(t) - \hbar\omega/2 & \Delta_0\Phi\hat{b} \\ \Delta_0\Phi\hat{b}^\dagger & -E(t) + \hbar\omega/2 \end{pmatrix}. \quad (3)$$

Within the conditions for adiabaticity outlined above, we can now analyze the probability for mechanically induced transitions between the Andreev levels as a function

of the coupling strength. As such transitions occur only in the vicinity of t_0, one can use the parabolic expansion of $E(t)$ around this point, which leads to the following time-dependent Schrödinger equation,

$$i\partial_\tau \vec{c}_n(\tau) = (\tau^2 \sigma_z + \Gamma\sqrt{n}\sigma_x)\vec{c}_n(\tau),$$

$$\tau = (t - t_0)(\xi/\hbar)^{1/3}, \qquad \Gamma = \frac{\Phi\Delta_0}{\hbar\omega}\left(\frac{V_c}{V}\right)^{2/3},$$

$$\xi = \frac{\partial^2 E(t)}{\partial t^2}\bigg|_{t_0} = \frac{D(\hbar\omega)^3}{\hbar^2}\left(\frac{V}{V_c}\right). \qquad (4)$$

Here, $\vec{c}_n(\tau) = [c_{+,n-1}(\tau), c_{-,n}(\tau)]^T$, where $c_{\pm,n}(\tau)$ is the probability amplitude for finding the system in the upper or lower electronic branch with the harmonic oscillator in the state n.

Within the adiabatic regime, where $V \lesssim V_c$, the coupling between the Andreev levels is relatively weak; $\Gamma = \Delta_0\Phi/(\hbar\omega) \ll 1$ (see below). This allows us to analyze (4) by treating Γ as a perturbation to the solution $|c_{-,n}(\tau)|^2 = 1$, where the system starts in the lower electronic branch at time $-\delta\tau = -\delta t(\xi/\hbar)^{1/3}$ and remains there until $+\delta\tau$. Solving (4) for the probability $|c_{+,n-1}(\delta\tau)|^2$ to find the system in the upper branch after passing through the resonance, we find that $|c_{+,n-1}(\delta\tau)|^2 \simeq \pi n\Gamma^2$, which is consistent with our numerical analysis.

In order to model the full evolution of the coupled electromechanical system we evaluate the total density matrix $\hat{\rho}$ of the system over one period. Under the assumption that the external temperature T is much smaller than the superconducting gap, the system will initially be found in the lower electronic branch with n quanta of the nanowire vibrations excited with probability P_n^{in},

$$\hat{\rho}^{\,in} = \sum_{i,j=\pm}\sum_{n=0}^{\infty} P_n^{in}|i,n\rangle\langle j,n| = \sum_{n=0}^{\infty} P_n^{in}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}|n\rangle\langle n|.$$

During the adiabatic evolution (no coupling between branches) the system interacts with the external heat bath and the rate of change of the density matrix is given by the expression

$$\partial\hat{\rho}(t)/\partial t = -i[\hat{\mathcal{H}}_{\rm eff}(t), \hat{\rho}(t)]/\hbar + \gamma\hat{\mathfrak{L}}(\hat{\rho}(t))/2, \quad (5)$$

$$\hat{\mathfrak{L}}(\hat{\rho}) = -(1 + n_B)(\hat{b}^\dagger\hat{b}\,\hat{\rho} + \hat{\rho}\hat{b}^\dagger\hat{b} - 2\hat{b}\,\hat{\rho}\,\hat{b}^\dagger)$$
$$- n_B(\hat{b}\hat{b}^\dagger\hat{\rho} + \hat{\rho}\,\hat{b}\,\hat{b}^\dagger - 2\hat{b}^\dagger\hat{\rho}\,\hat{b}), \quad (6)$$

where the collision integral $\hat{\mathfrak{L}}(\hat{\rho})$ models the interaction of the mechanical subsystem with the environment. Here, $n_B = [\exp(\beta\hbar\omega) - 1]^{-1}$ with $\beta = (k_B T)^{-1}$, while $\gamma = \omega/Q$ is the damping rate of the nanowire vibrations with Q the quality factor. Considering the coupling to the environment to be small ($Q \sim 10^5$ [18]) we can proceed by treating the interaction of the system with the external environment as a perturbation.

The large separation of the energy scales ensures that transitions between the Andreev levels is almost always negligible. It is only during a short time of order $\delta t \simeq (\hbar\omega\Phi/\Delta_0)^{1/2}\hbar/(eV) \ll \pi\hbar/(eV)$ around t_0 that the two branches interact, which permits us to describe the evolution of $\hat{\rho}$ through the resonance, $\hat{\rho}(t_0 + \delta t) = \hat{S}\hat{\rho}(t_0 - \delta t)\hat{S}^\dagger$, by the unitary scattering matrix $\hat{S}$,

$$\hat{S} = \begin{pmatrix} \kappa_1(\hat{n}) & i\frac{\nu_1(\hat{n})}{\sqrt{\hat{n}+1}}\hat{b} \\ i\hat{b}^\dagger\frac{\nu_2(\hat{n}+1)}{\sqrt{\hat{n}+1}} & \kappa_2(\hat{n}) \end{pmatrix}. \qquad (7)$$

In (7), $\hat{n} = \hat{b}^\dagger\hat{b}$ is the vibron number operator and the subscripts 1, 2 refer to the top and bottom Andreev level, respectively. The coefficient $\kappa_i(\hat{n})$ [$\nu_i(\hat{n})$] is the probability amplitude for the system to stay in [scatter out of] the initial state i, which depends on the state of the oscillator as outlined above [$|\kappa_i(n)|^2 + |\nu_i(n)|^2 = 1$]. As such, $|\nu_2(n)|^2$ is the probability of the system, initially in the lower electronic branch with n vibrons excited, to scatter into the upper electronic branch through the absorption of a vibron. It thus corresponds to $|c_{+,n-1}(\delta\tau)|^2$ in (4). We conclude that $|\nu_2(n)|^2 \simeq \pi n\Gamma^2$ and note that $|\nu_2(n)|^2 = |\nu_1(n-1)|^2$, from the symmetry of (4).

After one period the bound Andreev states merge with the continuum of itinerant states. At this time the quasiparticle carried by the Andreev states is released into the continuum and the initial conditions of the Andreev level populations are reset (see Fig. 2) [19]. The mechanical system will, however, not return to the initial vibrational state. Instead we must find its final state from the density matrix for the mechanical system after one period, $\hat{\rho}_{\rm mech}^f$, obtained by tracing out the electronic degrees of freedom in the total density matrix,

$$\hat{\rho}_{\rm mech}^f = {\rm Tr}_{\rm el}(\hat{\rho}^f)$$
$$= {\rm Tr}_{\rm el}\left[\hat{S}\hat{\rho}^{in}\hat{S}^\dagger + \frac{\gamma}{2}\frac{T_V}{2}\hat{S}\hat{\mathfrak{L}}(\hat{\rho}^{in})\hat{S}^\dagger\right.$$
$$\left. + \frac{\gamma}{2}\frac{T_V}{2}\hat{\mathfrak{L}}(\hat{S}\hat{\rho}^{in}\hat{S}^\dagger)\right]. \qquad (8)$$

Equation (8) describes the evolution of the mechanical density matrix over one period. Without coupling to the environment, $\gamma = 0$, this corresponds to a decay of the mechanical subsystem, since for each period there is a probability $|\nu_2(n)|^2$ that the state $|-, n\rangle$ scatters into the state $|+, n - 1\rangle$. As the opposite process is forbidden if initially only the lower electronic branch is populated, the mechanical subsystem would thus approach the vibrational ground state at a rate which depends on the strength of the coupling. Including effects of the environment, however, the mechanical subsystem will not reach its ground state as the external damping drives the system towards thermal equilibrium. In Fig. 3 we plot the evolution—over many periods starting from its initial thermal distribution—of the probability P_n for n quanta of mechanical nanowire vibra-
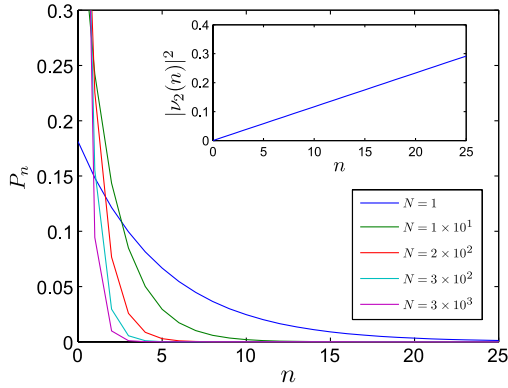
FIG. 3 (color online).   Distributions of the probabilities P_n for n quanta of the lowest flexural vibration mode of the suspended nanowire in Fig. 1 to be excited after N of the cooling cycles described in Fig. 2. Initially P_n is thermally distributed, $P_n = \exp(-\beta\hbar\omega n)[1 - \exp(-\beta\hbar\omega)]$, with $k_B T = 5\hbar\omega$. The plots were calculated for $\hbar\omega = 10^{-6}$ eV, $\Delta_0 = 10\hbar\omega$, $y_0 = 20$ pm, $L = 100$ nm, $H = 1$ T, and $T_V \sim 20$ ns. The inset shows $|\nu_2(n)|^2$ for the same parameters.

tions to be excited using realistic experimental parameters. As can clearly be seen, the scattering process acts to lower the number of excited quanta, effectively cooling the nanowire down to a final average vibron population $\langle n \rangle \sim 0.1$. This is confirmed by a perturbative calculation with $\Gamma \ll 1$, which shows that to first order in the damping rate γ the stationary population of the first excited state is

$$P_1^{\text{stat}} \simeq \frac{1}{Q\Phi^2} \frac{\hbar\omega}{\Delta_0} \left(\frac{V}{V_c}\right)^{1/3} \frac{1}{e^{\beta\hbar\omega} - 1} \sim 0.1, \qquad (9)$$

which is in accordance with the distribution in Fig. 3. Here we note that including a small initial population of the upper Andreev branch, $c_+^{\text{in}} \ll 1$, only changes the stationary vibron population to $\sim \langle n \rangle + O(c_+^{\text{in}})$. Thus, including, e.g., a thermal population of the upper Andreev branch only increases the stationary vibron population by a factor $c_+^{\text{in}} \simeq \exp(-2\beta\Delta_0) \sim 0.02$, and does not change the qualitative results of this Letter.

To conclude, we have shown that quantum mechanical cooling of a nanomechanical resonator acting as the weak link in a Josephson junction is possible. In particular, we have considered the example of a suspended carbon nanotube, where the unique combination of high resonance frequencies and high mechanical quality factors together with a high transparency for electrons combine to give sufficiently strong coupling for efficient cooling. Using realistic experimental parameters we have shown that for a short suspended nanotube, a stationary distribution of mechanical vibrational quanta corresponding to an average occupation number as low as $\langle n \rangle = 0.1$ can be achieved. This is truly in the quantum mechanical regime. Furthermore, the suggested mechanism does not rely on any external electromagnetic fields to stimulate the cooling process. Rather, the proposed system should act as a self-

cooling device as the "overcooled" Andreev states can readily absorb energy from the mechanical subsystem given that sufficient coupling between the two can be achieved. The corresponding energy uptake of the electronic subsystem is later released into the quasiparticle continuum, leading to an effective cooling of the nanomechanical resonator.

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[19] As explained by Gorelik *et al.* [Phys. Rev. Lett. **81**, 2538 (1998)], the nonequilibrium population of the Andreev states that approach the edges of the quasiparticle energy spectrum at $E = \pm\Delta_0$ (Fig. 2) cannot, due to selection rules, affect the population of the new Andreev states, which are created with populations reset to the thermal equilibrium and move away from the edges. This system will not be heated since (i) the released energy is carried away by quasiparticles [E. N. Bratus *et al.*, Phys. Rev. B **55**, 12 666 (1997)], which escape from the vicinity of the weak link and spread over the reservoirs with greatly reduced density, and (ii) the "escape time," $T_{\text{esc}} \sim (eV/\Delta_0)^{1/3}T_V$, is short compared to the period T_V.