Ground-State Cooling of a Suspended Nanowire through Inelastic Macroscopic Quantum Tunneling in a Current-Biased Josephson Junction

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We demonstrate that a suspended nanowire forming a weak link between two superconductors can be cooled to its motional ground state by a supercurrent flow. The predicted cooling mechanism has its origins in magnetic field induced inelastic tunneling of the macroscopic superconducting phase associated with the junction. Furthermore, we show that the voltage drop over the junction is proportional to the average population of the vibrational modes in the stationary regime, a phenomenon which can be used to probe the level of cooling.

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Nanoelectromechanical systems (NEMS) are fast approaching the limits set by quantum mechanics [1,2]. Achieving such conditions requires that the mechanical subsystem can be brought into, and detected in, its quantum mechanical ground state. This condition demands that an energy quantum associated with the mechanical motion is much larger than the energy associated with the thermal environment. For an oscillator with a mechanical frequency of 100 MHz this implies temperatures as low as a few mK. However, using oscillators with higher mechanical frequencies the quantum limit can be reached, as recently demonstrated by O'Connell *et al.* [3].

The most common device geometries of NEMS to date consist of mechanical oscillators in the form of cantilevers, suspended beams, or microtoroids. These typically have much lower resonance frequencies than those reported in Ref. [3]; hence, reaching the quantum limit with cryostatic techniques with these devices is challenging. To circumvent this, backaction cooling of the mechanically compliant element is often employed whereby the number of mechanical vibrons is reduced without necessarily lowering the ambient temperature. Suggestions for different cooling mechanisms are plentiful; see, e.g., Refs. [4–8].

In this Letter we suggest a cooling mechanism not previously considered. Considering the nanomechanical oscillator as a weak link in a current-biased Josephson junction, we show that we can access a regime analogous to the resolved sideband limit, whereby the number of mechanical vibrons can be reduced by a factor of ~100. In the limit of a high mechanical quality factor the resulting vibron population is shown to be well within the quantum regime. The cooling mechanism considered is achieved by coupling the mechanical vibrations of the oscillator to the supercurrent through the junction, Fig. 1. Below we show that the suggested setup not only allows for ground-state cooling of the mechanical oscillator, but simultaneously probes the macroscopic nature of the superconducting phase associated with the junction.

The classical description of the dynamics of the phase difference ϕ between the superconducting electrodes in a current-biased Josephson junction corresponds to that of a particle in a 2π-periodic potential tilted by the bias current I, the so-called tilted washboard potential, [9]. With the advent of smaller tunneling junctions, the possibility of quantum fluctuations of the superconducting phase was suggested and later experimentally confirmed; see, e.g., Ref. [10]. Within this description the quantum phase fluctuations can be described as tunneling transitions between quasibound states which are highly localized to the valleys of the washboard potential; see Fig. 2. Such phase fluctuations are significantly increased if the energy of two states in adjacent valleys coincide, under which conditions the tunneling has resonant character [11,12]. From a physical point of view this can be understood by considering that a fluctuation of the phase is accompanied by a fluctuation of the voltage over the junction. As a consequence, such fluctuations are associated with an energy release $\Delta E_I = hI/2e$ on the junction during the charge transfer between the leads. Under resonant conditions $\Delta E_I = \hbar\omega_p$ (ω_p is the plasma frequency) this energy can be absorbed by the electronic system in the form of a plasma excitation.

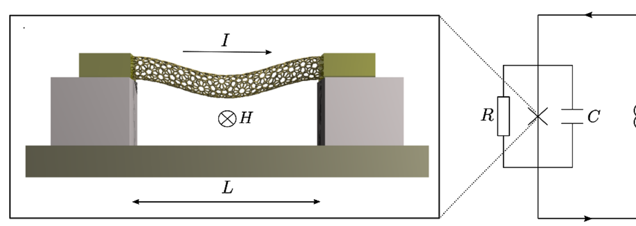

FIG. 1 (color online). Schematic diagram of the system. Left: A suspended nanowire of length L forms a weak link between two current-biased superconducting leads. The transverse magnetic field H is applied perpendicular to the nanowire. Right: The equivalent electronic circuit. A constant current I is applied to the Josephson junction which is connected in parallel to a capacitor C and a resistance R.

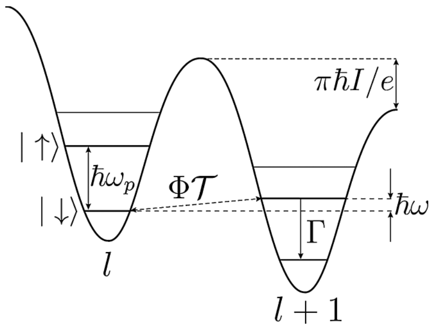

FIG. 2. Schematic diagram of the tilted washboard potential $U(\phi) = -E_J \cos\phi - j\hbar\phi$, as a function of phase ϕ at current bias $I = e/\pi(\omega_p - \omega)$. Here, l labels the valleys of the potential and $\sigma = \uparrow, \downarrow$ are the two energy levels within the valleys considered. In the above, $\Phi\mathcal{T}$ is the inelastic tunneling amplitude between two energy levels in consecutive valleys. The quantity Γ is the transition rate from the second to the first level within a valley generated by interactions with the quasiparticle environment (see text).

If the Josephson junction is coupled to a mechanical oscillator with eigenfrequency $\omega \ll \omega_p$, the mechanical subsystem can be involved in the resonant transitions if $\Delta E_I = \hbar\omega_p \pm \hbar\omega$. Here, we show that this might be used to achieve ground-state cooling of the mechanical oscillator. By tuning the bias current to satisfy $\Delta E_I = \hbar\omega_p - \hbar\omega$, the system, initially prepared in the ground state of a given valley, can coherently tunnel to the first excited level in the adjacent valley with the absorption of a mechanical quantum. After this transition the system may either tunnel back with the emission of a vibron or incoherently decay to the ground state in the same valley; see Fig. 2. In the latter case, the electronic subsystem returns to its original configuration, whereas the energy of the mechanical subsystem is reduced. Repeating this process, the mechanical oscillator can be brought to its motional ground state, under which conditions further absorption transitions are blocked. If, however, an external thermostat excites the mechanical subsystem, it will stimulate further inelastic transitions with the associated potential drop over the junction, after which the mechanical subsystem is brought back to the ground state.

Figure 1 shows the system considered, which consists of a metallic carbon nanotube suspended over two superconducting leads biased at a current I. Transverse to the in-plane motion of the nanotube a magnetic field H is applied which induces coupling between the bending modes of the wire to the supercurrent through it. The Hamiltonian describing the system in Fig. 1 has the form

$$\hat{\mathcal{H}} = 4E_c\hat{n}^2 - j\hbar\hat{\phi} - E_J\cos(\hat{\phi} - \Phi\hat{u}) + \hbar\omega\hat{b}^\dagger\hat{b}. \quad (1)$$

Here, $\hat{n}$ is the operator for the number of Cooper pairs on the junction and $\hat{\phi}$ is the corresponding operator for the superconducting phase ($[\hat{\phi}, \hat{n}] = i$) [13]. In (1), $E_c = e^2/(2C)$ is the Coulomb energy where C is the capacitance of the junction, $j = I/(2e)$ is the flow of Cooper pairs, and E_J is the Josephson energy [14].

In what follows we limit the description of the mechanical degrees of freedom to the fundamental bending mode, which is considered as a harmonic oscillator with frequency ω. In (1), the operators $\hat{b}^\dagger$ [$\hat{b}$] are creation [annihilation] operators for the oscillator where $\hat{u} = \hat{b} + \hat{b}^\dagger$ is the deflection of the wire. The parameter $\Phi = 4g\pi LHu_{\rm zp}/\Phi_0$ defines the coupling strength between the mechanical and electronic degrees of freedom. Here, $u_{\rm zp} = [\hbar/(2m\omega)]^{1/2}$ is the zero-point amplitude of the nanowire, m and L are the effective mass and length of the suspended part of the wire, respectively, $\Phi_0 = \pi\hbar/e$ is the flux quantum, and g is a factor of order of unity which accounts for the profile of the fundamental mode [15].

The third term in (1) describes the influence of the electromechanical coupling, which arises due to the induced electromotive force caused by the motion of the current-carrying wire in the magnetic field [16]. This term describes, on the one hand, the Lorentz force on the nanowire induced by the Josephson current. On the other hand, it gives the deflection dependence of the Josephson current due to the motion of the wire in the magnetic field. In what follows we consider a nanotube of length $L = 1\ \mu$m, frequency $\omega = 10^8\ {\rm s}^{-1}$, and effective mass $m = 0.4 \times 10^{-21}$ kg [17] in a magnetic field $H = 1$ T. With these parameters $u_{zp} \simeq 35$ pm and $\Phi \lesssim 0.3$; hence, we consider only the linear terms in the expansion of (1) with respect to Φ. Thus, we identify the Josephson Hamiltonian $\hat{\mathcal{H}}_J = 4E_c\hat{n}^2 - j\hbar\hat{\phi} - E_J\cos(\hat{\phi})$, which under the condition $j < E_J/\hbar$ describes the electronic subsystem in the tilted washboard potential, Fig. 2. With this expansion the interaction between the electronic and mechanical subsystem is $\hat{\mathcal{H}}_{\rm int} = -E_J\Phi(\hat{b}^\dagger + \hat{b}) \times \sin(\hat{\phi})$, whereas the mechanical Hamiltonian is $\hat{\mathcal{H}}_m = \hbar\omega\hat{b}^\dagger\hat{b}$.

Below we take the Coulomb energy to be much smaller than the Josephson energy, $4E_c/E_J \ll 1$. This implies that the characteristic interlevel distance between the two lowest quantized states of the Josephson junction associated with a local minimum of the potential, $\hbar\omega_p = (8E_JE_c)^{1/2}$, is much smaller than the height of the barrier separating different local minima. We also take the external temperature T to be low, $T \ll \hbar\omega_p/k_B$, such that transitions between states in different local minima can only occur through underbarrier tunneling, a phenomenon commonly referred to as macroscopic quantum tunneling (MQT). A schematic diagram of the quantum state of the electronic subsystem is shown in Fig. 2.

With the above energy quantization we define the critical bias current I^* which ensures that the lowest (first) level is resonant with the second level in the next valley, $I^* \simeq e\omega_p/\pi$ [12]. Note that the potential defined by $\hat{\mathcal{H}}_J$ is only to first approximation parabolic, which implies the spacing between the energy levels within a given valley is not constant. As such, we will in the following only consider tunneling between the two lowest electronic states and neglect any coupling to higher levels. This is justified as, e.g., the second and third levels are far from resonance if the junction is biased at $I \simeq I^*$ (see Fig. 2).

The electromechanical coupling induced by the magnetic field implies that MQT can in the present situation also be accompanied by emission or absorption of a quantum of mechanical energy. Performing a WKB analysis for the MQT amplitude we find that the overlap integrals for these inelastic channels are of the order of $\Phi\mathcal{T}$, where $\mathcal{T} \propto \hbar\omega_p \exp[-\pi(E_J/(2E_c)^{1/2}] < \hbar\omega$ is the tunneling amplitude in the elastic channel. Here, we note that the ϕ dependence of $\hat{\mathcal{H}}_{\rm int}$ only leads to a renormalization of the parameter g in the definition of Φ. Also note that due to the large separation in energy, $\omega \ll \omega_p$, the electromechanical coupling will not introduce additional tunneling channels between the higher electronic energy levels.

Tunneling through the inelastic tunneling changes the number of mechanical vibrons such that cooling of the oscillator is possible as outlined above. Below we show that this can be achieved by tuning the bias current so that the absorption channel is resonant; the first level in a valley l is separated by $\hbar\omega$ from the second level in $l+1$ as shown in Fig. 2. A further condition for cooling is that the electronic subsystem, once in the second energy level, relaxes to the lower level at a rate Γ, which is faster than the rate at which the system tunnels back with the emission of a vibron, $\Gamma > \mathcal{T}/\hbar$. Such relaxation arises due to interaction with the quasiparticle environment, as discussed further below [18].

To perform a quantitative analysis of the system we introduce the basis $|l,\sigma\rangle$ where $\sigma = \uparrow,\downarrow$ labels the energy levels inside a given valley ($\downarrow$ is the first and $\uparrow$ is the second level). In this basis the Hamiltonian reads

$$\hat{\mathcal{H}} = \hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\mathcal{T}},$$
$$\hat{\mathcal{H}}_0 = \hat{\mathcal{H}}_J + \hat{\mathcal{H}}_m = \sum_{l,n,\sigma}(\mathcal{F}_{l,\sigma} + \hbar\omega\hat{b}^\dagger\hat{b}|l,\sigma\rangle\langle l,\sigma|,$$
$$\hat{\mathcal{H}}_{\mathcal{T}} = \sum_l \mathcal{T}(\Phi(\hat{b}+\hat{b}^\dagger)+1)|l+1,\uparrow\rangle\langle l,\downarrow| + \text{H.c.} \quad (2)$$

In the above, $\mathcal{F}_{l,\sigma} = \hbar\omega_p m_\sigma - l\pi\hbar I/e$ are the eigenvalues for the electronic degrees of freedom in the basis $|l,\sigma\rangle$, where $m_\uparrow = 1$ and $m_\downarrow = 0$. From the form of the Hamiltonian (2) one can see that due to the electromechanical coupling the number of vibrons in the system is not conserved and may change due to macroscopic tunneling of the electronic system from one valley to the next.

To describe the joint dynamics of the electronic and mechanical degrees of freedom, we will start our analysis from the Liouville–von Neumann equation for the density matrix $\hat{\rho}$ of the system,

$$\frac{\partial\hat{\rho}}{\partial t} = -\frac{i}{\hbar}[\hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_{\mathcal{T}}, \hat{\rho}] + \hat{J}(\hat{\rho}) + \gamma(1+n_B)\mathcal{L}_{\hat{b}}(\hat{\rho}) + \gamma n_B \mathcal{L}_{\hat{b}^\dagger}(\hat{\rho}). \quad (3)$$

Here, $\hat{J}(\hat{\rho})$ is a phenomenological damping operator for the electronic system [12],

$$\hat{J}(\hat{\rho}) = -\frac{\Gamma}{2}(\sum_l |l,\uparrow\rangle\langle l,\uparrow|\hat{\rho} + \hat{\rho}|l,\uparrow\rangle\langle l,\uparrow|) + \Gamma\sum_{l,l'}|l,\downarrow\rangle\langle l,\uparrow|\hat{\rho}|l',\uparrow\rangle\langle l',\downarrow|. \quad (4)$$

In the equivalent circuit scheme (see Fig. 1) this term derives from the parallel resistance R, which in the present situation causes the system to decay from the second ($\uparrow$) to the first ($\downarrow$) level in a given valley. In (4), $\Gamma = \omega_p/Q_{\rm el}$ is the electronic damping rate, where $Q_{\rm el} = \omega_p RC$ is the corresponding quality factor. Here we consider $Q_{\rm el} \gg 1$, which implies that the influence from the electronic quasiparticle environment on the tunneling processes is negligible [12,19,20]. We will further suppose that the quality factor $Q_{\rm el}$ is so large that broadening of the second energy level, $\Delta\omega_p = \omega_p/(2Q_{\rm el})$, is small enough for the inelastic resonance transitions to be resolved, $\Delta\omega_p < \omega$.

The second damping term in (3), $\mathcal{L}_{\hat{a}}(\hat{\rho}) = (2\hat{a}\,\hat{\rho}\,\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\,\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a})/2$, is the standard Lindblad operator which models interactions between the oscillator and the thermal environment. Here, $\gamma = \omega/Q$ is the mechanical damping rate with Q the quality factor and $n_B = [\exp(\beta\hbar\omega) - 1]^{-1}$, where $\beta = (k_B T)^{-1}$ is the average number of vibrons in thermal equilibrium.

Below we investigate the stationary solution to (3). To find this solution we perform a standard perturbative analysis in the small parameters $\mathcal{T}/(\hbar\Gamma)$, $\gamma/\Gamma \propto \epsilon \ll 1$, and look for a solution of the density matrix of the form $\hat{\rho} = \hat{\rho}_0 + \epsilon\hat{\rho}_1 + \epsilon^2\hat{\rho}_2 + \cdots$ (for a full derivation of the results presented below, see [21]). Substituting this into (3) one finds that the leading order solution $\hat{\rho}_0$ has the form $\hat{\rho}_0 = \sum_{l,n}|l,\downarrow,n\rangle\rho_0(l,\downarrow,n)\langle l,\downarrow,n|$, where the index n labels the Fock state of the oscillator. From (3) we also calculate the first order correction $\hat{\rho}_1 = \sum_{l,n,j=-1,0,1}|l+1,\uparrow,n+j\rangle\times c_j(l,n)\langle l,\downarrow,n| + \text{H.c.}$ With this we find that the equation for the second order term, $\hat{\rho}_2$, can only be resolved if the coefficients $P(n) = \sum_l \rho_0(l,\downarrow,n)$—which give the population of the vibrational modes of the oscillator—satisfy the following equation:

$$[\Gamma_- + \gamma(1+n_B)][(n+1)P(n+1) - nP(n)] + (\Gamma_+ + \gamma n_B)[nP(n-1) - (n+1)P(n)] = 0. \quad (5)$$

Here, Γ_j are the tunneling rate, and $j = -, 0, +$ are, respectively, the absorption, elastic, and emission channel,

$$\Gamma_\pm = \Gamma\frac{4\Phi^2\mathcal{T}^2}{4(\Delta\mathcal{F}_\pm)^2 + \hbar^2\Gamma^2}, \quad \Gamma_0 = \Gamma\frac{4\mathcal{T}^2}{4(\Delta\mathcal{F}_0)^2 + \hbar^2\Gamma^2},$$
$$\Delta\mathcal{F}_0 = \mathcal{F}_{l+1,\uparrow} - \mathcal{F}_{l,\downarrow}, \quad \Delta\mathcal{F}_\pm = \mathcal{F}_0 + \hbar\omega.$$

Considering the operator for the potential drop over the Josephson junction $\hat{V} = i[\hat{\mathcal{H}}, \hat{\phi}]/(2e)$ (in our representation $\hat{\phi} = 2\pi\sum_{l,\sigma}|l,\sigma\rangle l\langle l,\sigma|$), we find

$$\hat{V} = \frac{\pi}{ie}\sum_{l}\mathcal{T}(\Phi(\hat{b}+\hat{b}^{\dagger})+1)|l+1,\uparrow\rangle\langle l,\downarrow| + \text{H.c.} \quad (6)$$

This implies that the bias voltage, $V = \text{Tr}(\hat{V}\hat{\rho})$, is zero (the phase is stationary) to leading order in $\hat{\rho}$, and that the potential drop is given by the first order correction to the density matrix, $V = \text{Tr}(\hat{V}\hat{\rho}_1)$. Solving (5) we find that the analytic solution for the average number of vibrons, $\langle n\rangle = \sum_n nP(n)$, and bias voltage are given by

$$\langle n\rangle = \frac{n_B\gamma + \Gamma_+}{\gamma + \Gamma_- - \Gamma_+}, \quad (7)$$

$$V = \frac{\pi\hbar}{e}(\Gamma_-\langle n\rangle + \Gamma_0 + \Gamma_+(\langle n\rangle + 1)). \quad (8)$$

The potential drop in the stationary regime is primarily determined by the elastic tunneling rate, Γ_0. This is consistent with the physical processes discussed; i.e., in the limit $\gamma, \Gamma_+ \to 0$, we get $\langle n\rangle = 0$ (complete ground-state cooling as no heating channel is open) and $V \propto \Gamma_0$ (the system moves down the potential at the rate Γ_0 which conserves the number of vibrons). Note, however, that Γ_0 does not enter into the analytic solution of $\langle n\rangle$ as MQT through the elastic channel does not change the number of mechanical vibrons.

In Fig. 3 we plot both the average stationary population of the mechanical subsystem and the corresponding voltage drop as a function of the bias current. As expected, the lowest occupation is achieved when $I = I^* - e\omega/\pi$ (see Fig. 2). In this regime, we find that ground-state cooling of the mechanical subsystem is possible if the resolved sideband limit, $\omega > \Gamma$, is achieved. Under conditions when the bias current is $I > I^*$, the tunneling events discussed above will lead to pumping of the mechanical subsystem, in which case the above analysis does not apply once the limit $\mathcal{T}(\langle n\rangle + 1) \sim \hbar\Gamma$ is reached. This regime will be discussed in future work.

To conclude, we have shown that a suspended nanowire weak link in the current-biased Josephson junction can be cooled to its motional ground state. This effect derives from the electromechanical coupling generated by the

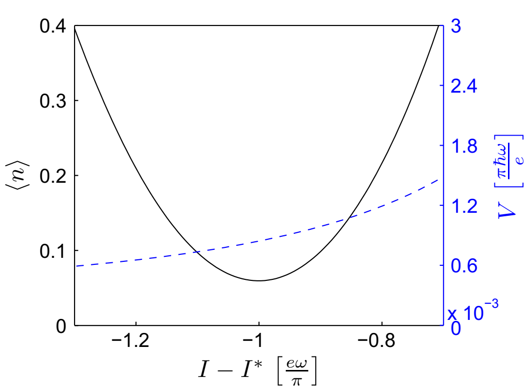

FIG. 3 (color online). Average vibron population (solid line) and bias voltage (dashed line) in the stationary regime as a function of the current bias. Here, $\Phi = 0.3$, $\Gamma = \omega/4$, $\mathcal{T} = \hbar\omega/20$, $n_B = 20$, and $Q = 10^5$.

magnetic field, which opens inelastic transition channels in the electronic subsystem. Our analysis shows that the occupation factor of the vibrational modes can be tuned by the bias current, and that ground-state cooling is possible if the resolved sideband limit can be achieved. The associated potential drop over the junction might be a sensitive probe of the mechanical subsystem as it scales with the average number of vibrons.

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