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Current modulation of nanoconstriction spin Hall nano-oscillators

Mohammad Zahedinejad¹, Ahmad A. Awad¹, Philipp Dürrenfeld^{3,1}, Afshin Houshang¹, Yuli Yin^{4,1},

P. K. Muduli⁵, and Johan Åkerman^{1,2}

¹Department of Physics, University of Gothenburg, 412 96 Gothenburg, Sweden

²Materials and Nanophysics, School of ICT, KTH Royal Institute of Technology, 164 00 Kista, Sweden

³School of Electronic Science and Engineering, Nanjing University, 210093 Nanjing, China

⁴Department of Physics, Southeast University, 211189, Nanjing, China

⁵Department of Physics, Indian Institute of Technology, 110016, New Delhi, India

A single nanoconstriction spin Hall oscillator (NC-SHNO) in out-of-plane fields is presented as a non-linear amplitude and frequency modulator operated by radio frequency (RF) current modulation. The current modulation was carried out in different NC-SHNO nonlinearity regimes corresponding to negative, zero, and positive values of df/dI, in order to investigate the device response to an 80 MHz modulating current. Our study showed that current modulation of SHNOs can be quantitatively predicted by a nonlinear frequency and amplitude modulation (NFAM) model using the values of df/dI and d^2f/dI^2 extracted from the free-running f vs. I profile. The NFAM model reproduces the asymmetric sideband amplitude as well as the red and blue shift of the frequency in excellent agreement with the experimental results. The ability to predict the modulation process is a necessary benchmark in designing SHNO modulators for future integrated microwave circuits.

Index Terms-Spintronics, spin Hall nano-oscillator, non-linear frequency and amplitude modulation.

I. INTRODUCTION

Spin transfer torque (STT) [1], [2], [3] devices hold great promise for a wide range of applications [4], such as magnetic memory [5], [6], [7], [8], microwave assisted magnetic recording [9], [10], microwave signal generation [11], [12] and detection [13], spin wave generation [14], [15], [16], [17], [18], [19], [20], high frequency modulators [21], [22], and, more recently, in neuromorphic computing [23], [24], [25]. STT devices operate through the transfer of angular momentum from a spin-polarized current to the local magnetization, which can fully compensate the local spin wave damping and lead to spin wave auto-oscillations. Through the magnetoresistance of the device, these auto-oscillations can then generate an electrically tunable microwave voltage in devices known as spin torque nano-oscillators (STNOs) [26], [27], [28], [4].

Recently, a new class of STT devices, called spin Hall nanooscillators [29], [30], [31], [32], [33], [34] (SHNOs) were realized, in which the spin Hall effect [35] (SHE) from a nonmagnetic (NM) layer with large spin-orbit coupling drives a pure spin current into an adjacent ferromagnetic (FM) layer. Nano-constriction based SHNOs are particularly attractive as their fabrication only requires a NM/FM bilayer and a single lithography step. Their device geometry also provides for direct optical access of the auto-oscillating regions, and when connected in series, can exhibit robust mutual synchronization [36], [37].

In order to use SHNOs for communication applications, it is required that their microwave signal can be modulated. In this letter, we demonstrate efficient modulation of a nanoconstriction SHNO via its drive current. We study its modulation properties at different operating points characterized by different sign and magnitude of its non-linear frequency and amplitude coefficients, and find that the non-linear frequency and amplitude modulation (NFAM) model [38], [39] accurately predicts the SHNO modulation performance based on its un-modulated behavior.

II. FABRICATION AND MEASUREMENT TECHNIQUE

The SHNO stack consisted of a 5 nm Pt and 5 nm Py (Ni₈₀Fe₂₀) bilayer, and a 5 nm SiO₂ protective layer, all magnetron sputtered at room temperature onto 20×20 mm² C-plane sapphire substrates using a 3 mTorr argon plasma pressure in an AJA sputtering chamber with a 10^{-8} Torr base pressure. 80–200 nm wide nano-constrictions were then fabricated using electron beam lithography and Argon ion beam dry etching. Coplanar wave guide (CPW) structures were defined using conventional optical lithography followed by Cu/Au deposition and lift-off.

Fig. 1(a) depicts the measurement setup. The direct drive current was applied through a microwave bias tee. The SHNO microwave signal was transmitted via the high-frequency port of the bias tee and subsequently amplified using a low noise +56 dB (4-10 GHz) amplifier before being recorded using a spectrum analyzer. A microwave circulator (4-12 GHz) was also connected to the high frequency port of the bias tee in order to inject the 80 MHz RF modulating current from a signal source into the free-running SHNO in addition to the direct current. The circulator insertion loss was measured to be 3.9 dB at the frequency of the modulating current using a network analyzer. A SHNO with a 200-nm constriction width was used in all measurements. The magnetic field was applied at an out-of-plane angle of θ =80° and an in-plane angle of $\varphi=23^{\circ}$, defined in the inset of Fig. 1(b). The strength of the applied magnetic field was kept constant at 0.76 T.

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III. RESULTS AND DISCUSSION

Fig. 1(b) shows the power spectral density (PSD) vs. applied direct current (I_{dc}) of the free-running SHNO in a tilted out-ofplane field, with the characteristic non-monotonic current dependence of its microwave frequency (f) [37], [40]. Fig. 1(c) shows the corresponding current dependence of the integrated power, extracted using Lorentzian fits to the PSD, including error bars.

Fig. 1(d) shows d^2f/dI^2 vs. I_{dc} after some smoothing of the experimental data in Fig. 1(b). For our modulation experiments, we chose four different direct currents as shown by the dashed lines in Fig. 1(d).

In the NFAM model [38], [39], it is assumed that the instantaneous frequency depends non-linearly on the modulating signal $m(t) = I_m \sin(2\pi f_m t)$ as:

$$f_i(t) = \sum_{h=0}^{v} k_h m^h(t)$$
 (1)

where I_m is the modulating current amplitude, f_m is the modulating frequency, k_h is the h^{th} -order frequency sensitivity coefficient, and k_0 is the unmodulated SHNO frequency. The NFAM model also assumes that the instantaneous amplitude depends non-linearly on the modulation as:

$$A_c(t) = \sum_{l=0}^{u} \lambda_l m^l(t)$$
⁽²⁾

Here, the λ_l specifies the l^{th} -order amplitude sensitivity coefficient. The coefficients k_h and λ_l are calculated by fitting polynomials to f vs. I_{dc} and P vs. I_{dc} of the freerunning oscillator; in our case we chose fifth and third order polynomials for f vs. I and P vs. I, respectively. The output voltage amplitude spectrum predicted by the NFAM model can then be represented as follows:

$$S(f) = \frac{1}{4} \sum_{h=0}^{3} \gamma_h \sum_{n,m,p,q,r=-\infty}^{\infty} J_m(\beta_1) J_n(\beta_2) J_p(\beta_3) J_q(\beta_4) J_r(\beta_5) \times \left\{ \delta[f - f_c - (n + 2m + 3p + 4q + 5r + h)f_m] + \delta[f - f_c - (n + 2m + 3p + 4q + 5r - h)f_m] + \delta[f + f_c - (n + 2m + 3p + 4q + 5r - h)f_m] + \delta[f + f_c - (n + 2m + 3p + 4q + 5r - h)f_m] \right\}$$

where, $\beta_1 = k_1 I_m / f_m + 3k_3 I_m^3 / 4f_m$, $\beta_2 = k_2 I_m^2 / 4f_m + k_4 I_m^4 / 4f_m$, $\beta_3 = k_3 I_m^3 / 12f_m$, $\beta_4 = k_4 I_m^4 / 32f_m$, $\beta_5 = k_5 I_m^5 / 80$ stand for the frequency indices of different orders, while $\gamma_0 = \lambda_0 + \lambda_2 I_m^2 / 2$, $\gamma_1 = \lambda_1 I_m + 3\lambda_3 I_m^3 / 4$, $\gamma_2 = \lambda_2 I_m^2 / 2$, $\gamma_4 = \lambda_3 I_m^3 / 4$ are the amplitude modulation indices of different orders.

The frequency also depends non-linearly on the modulation:

$$f_c = k_0 + k_2 I_m^2 + 3k_4 I_m^4 / 8 + \dots$$
(4)

Knowing k_i and λ_i from the free-running behavior of the SHNO, the NFAM model can thus predict both f_c and the amplitudes of all sidebands located at $\pm sf_m$ relative to f_c . Here $s = n + 2m + 3p + 4q + 5r \pm h$ is an integer identifying the sideband order.



Fig. 1. (a) Schematic of the measurement setup. (b) Color map of the power spectral density (dB over noise) vs. current of the free-running SHNO in an out-of-plane field configuration with φ =23° and θ =80°. Inset shows schematic of the SHNO with definitions of φ and θ . (c) Integrated power vs. current. Inset: plot of the inverse integrated power vs. current, together with a linear fit at low currents (red line). (d) dI^2/dI^2 vs. current. The four operating points are marked by vertical dash black lines.

Four different operating points were selected to experimentally investigate current modulation of the SHNO and its effect on the magnetization dynamics: Three (3.9 mA, 4.13 mA, and 4.4 mA) with positive d^2f/dI^2 (upward concave), having a negative, zero, and a positive df/dI, respectively, and a fourth (4.75 mA) with negative d^2f/dI^2 (downward concave). It is worth noting that the maximum current (including RF current) that the device can withstand without failure is limited, due to the finite heat dissipation of the substrate and electromigration of the metallic bilayer in the constriction region. This in turn



Fig. 2. Modulation of the SHNO at the four different dc bias currents marked in Fig. 1(d): (a) 3.9 mA, (b) 4.1 mA, (c) 4.4 mA and (d) 4.75 mA. The first three operating points correspond to a positive df^2/dI^2 , while the dc current of 4.75 mA corresponds to a negative df^2/dI^2 . The white hollow circles are calculations based on the NFAM model.

leads to a decrease of the maximum allowed RF current when the I_{dc} operating point is high.

Fig. 2 shows the experimental results (color maps) when modulating the SHNO at the four operating points. The modulation is characterized by the appearance of multiple sidebands with increasing modulation current. During modulation, a frequency shift is also observed. The frequency shifts upwards (blue shift) for $I_{dc} = 3.9$, 4.13 and 4.4 mA while it shifts downwards (red shift) for $I_{dc} = 4.75$ mA, consistent with the sign of $d^2 f/dI^2$. This behavior can be well reproduced by the NFAM model (white hollow circles). The frequency shift is calculated directly from Eq. (4) using the second order frequency sensitivity coefficients shown in Table I.

Fig. 2 also indicates that the power of the upper and lower sidebands are not identical. In order to investigate the evolution of the carrier and sideband powers, we plot the integrated power for the carrier signal (CS), the first lower (LSB) and upper (USB) sidebands as a function of modulation current in Fig. 3, for the first three operating points in Fig. 2. The behavior of the CS, LSB, and USB powers can again be quantitatively reproduced by the NFAM model using Eq. (3), as shown by the solid lines. In particular, the opposite sign of k_1 for the cases of 3.9 and 4.4 mA, leads to the LSB having higher power than the USB for 3.9 mA (large negative k_1), whereas the opposite is true at 4.4 mA (large positive k_1). The much smaller k_1 for 4.13 mA leads to a much later onset of modulation, and the LSB and USB powers are determined by a combination of the higher order coefficients. By knowing the free-running behavior of the SHNO we can hence predict its modulation behavior in all cases. The predictable modulation behavior together with the easy fabrication hence make SHNOs highly promising for communication applications. The fact that the NFAM model provides an accurate quantitative



Fig. 3. Integrated power vs. modulation current of the CS, LSB, and USB at three dc operating points: (a) 3.9 mA, (b) 4.13 mA, and (c) 4.4 mA. The solid lines are fits based on the NFAM model using parameters from the un-modulated SHNO.

description of the modulated behavior also provides bounds of at least 80 MHz and at most 4 ns⁻¹ for the SHNO modulation bandwidth (f_p) and its amplitude relaxation rate ($\Gamma_p = \pi f_p$), respectively [21], [22], [26].

IV. CONCLUSION

In conclusion, current modulation of nano-constriction based spin Hall oscillators has been presented. Using modulation sensitivity coefficients obtained from free running oscillator, we find that the nonlinear frequency and amplitude modulation model (NFAM) can precisely predict the modulated behavior for both frequency shift and sideband amplitudes. The nonlinear behavior of the SHNO allows us to consider it as flexible modulator for use in future communication systems.

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TABLE I

MODULATION SENSITIVITY COEFFICIENTS EXTRACTED FROM FITTED POLYNOMIALS TO THE FREE-RUNNING BEHAVIOR OF f vs. I and P vs. I.

Current(mA)	$k_0(GHz)$	k_1 (GHz/mA)	$k_2(\text{GHz/mA}^2)$	k_3 (GHz/mA ³)	$k_4(\text{GHz/mA}^4)$	$k_5(\text{GHz/mA}^5)$	$\lambda_0(pW^{1/2})$	$\lambda_1(\mathbf{pW}^{1/2}/mA)$	$\lambda_2(\mathbf{pW}^{1/2}/mA^2)$	$\lambda_3(\mathrm{pW}^{1/2}/mA^3)$
3.9	8.5590	-0.2099	0.4737	-0.0007	0.2287	-0.0716	2.3281	0.9725	0.8116	3.5954
4.13	8.5357	-0.02501	0.5789	0.2442	-0.3345	-0.9367	2.6545	1.6235	1.9751	2.6632
4.4	8.5847	0.3107	0.3276	-0.6333	-0.1591	0.6520	3.4212	3.5014	0.4954	-4.8189

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