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Laplace processes for describing road profiles

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Abstract

The Gaussian model is frequently used for modelling environmental loads, e.g. sea elevation, wind loads and road profiles. However, the Gaussian model is often only valid for short sections of the load. For example, for roads profiles, short sections of roads, say 100 m, is well modelled by a Gaussian process, whereas longer sections of roads, say 10 km, typically contain shorter sections with high irregularity, and the variability between sections is higher than what can be explained by the stationary Gaussian model. This phenomenon can be captured by a Laplace process, which can be seen as a Gaussian process with randomly varying variance. Thus, the Gaussian process is a special case of the Laplace process. Further, the expected damage can be computed from the parameters of the Laplace process. We will give examples of modelling road profiles using Laplace models. Especially, it will be demonstrated how to reconstruct a road profile based on sparse road roughness measurements, such as a sequence of IRI (International Roughness Index) for 100 metre road sections. Further, IRI data from the Finnish road network will be evaluated.

Keywords: Road surface profile, road roughness, road irregularity, Laplace process, non-Gaussian process, power spectral density (PSD), ISO spectrum, roughness coefficient, international roughness index (IRI), vehicle durability, fatigue damage

1. Introduction

Durability assessment of vehicle components often requires a customer or market specific load description. It is therefore desirable to have a model of the load environment that is vehicle independent and which may include many factors, such as encountered road roughness, hilliness, curvature, cargo loading, driver behaviour and legislation.

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Here we consider modelling of the road surface roughness with focus on fatigue life prediction. Especially, we focus on reconstruction of road profiles based on measurements of the so-called International Roughness Index (IRI), which is often available from road administration data bases.

Traditionally, road profiles have been modelled by using Gaussian processes; see e.g. Dodds & Robson [6], ISO 8608 [8], and Andrén [1]. However, it is well known that measured road profiles contain shorter segments with above average irregularity, which is a property that cannot be modelled by a Gaussian process, and therefore several approaches has been suggested, see e.g. Bogsjö [2] and the references therein. In Bogsjö et al. [3] a new class of random processes, namely Laplace processes, has been proposed for modelling road profiles. Simply speaking it is a Gaussian process where the variance is randomly changing. A similar approach has been taken by Charles [5], Bruscella et al. [4], and Rouillard [12], [13].

In the case when only IRI data is available, a simple enough model is required in order to be able to estimate the model parameters. Therefore, we will use the non-stationary Laplace model presented in Bogsjö et al. [3], Johannesson & Rychlik [9], together with the standardized spectrum according to ISO 8608 [8], which gives a Laplace model with only two parameters to be estimated, where the first parameter describes the mean roughness, while the second parameter describes the variability of the variance which is the gamma distributed. In the non-stationary Laplace model the variance is constant for short segments of fixed length, typically one or some hundred metres. By using IRI data from the Finnish road network, we will demonstrate how to efficiently estimate the Laplace parameters, and evaluate their dependency on the type of road. Further, we will employ a simple but accurate approximation of the fatigue damage due to Laplace roads.

2. Road spectra and roughness coefficient

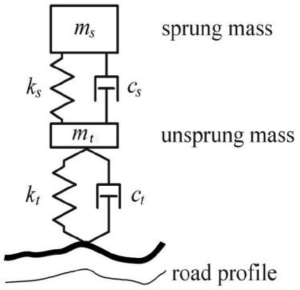
For stationary loads, power spectra are often used to describe the frequency content of a signal. The vertical road variability consists of the slowly changing landscape (topography), the road surface unevenness (road roughness), and the high variability components (road texture). For fatigue applications, the road roughness is the relevant part of the spectrum. Often one assumes that the energy for frequencies < 0.01 m^{-1} (wavelengths above 100 metres) represents landscape variability, which does not affect the vehicle dynamics and hence can be removed from the spectrum. Similarly high frequencies > 10 m^{-1} (wavelengths below 10 cm) are filtered out by the tire and thus are not included in the spectrum. ISO 8608 standard [8] uses a two parameter spectrum to describe the road profile $Z(x)$

$$S_Z(\Omega) = C\left(\frac{\Omega}{\Omega_0}\right)^w, \quad 2\pi\cdot 0.011 \le \Omega \le 2\pi\cdot 2.83 \text{ rad/m}, \quad \text{and zero otherwise} \tag{1}$$

where Ω is the spatial angular frequency, and $\Omega_0 = 1\ \text{rad/m}$. The spectrum is parameterized by the degree of unevenness C, here called the roughness coefficient, and the waviness w. The ISO spectrum is often used for quite short road section (in the order of 100 metres). For road classification the ISO standard uses a fixed waviness w = 2. Here we will use this simplified ISO spectrum, having only one parameter, namely the roughness coefficient C. The simplicity of the ISO spectrum makes it attractive to use in vehicle development, but it will not accurately approximate the road roughness spectrum for the whole range of frequencies, however, what is important is that they correctly estimate the energy for frequencies in the range which may excite the vehicle response, which obviously also depends on the vehicle speed. The choice of the ISO spectra is motivated by its simplicity, as it depends on only one parameter that can be related to IRI, as will be explained below.

3. International Roughness Index

When monitoring road quality, segments of measured longitudinal road profiles are often condensed into a sequence of IRI values, see Gillespie et al. [7]. They are calculated using a quarter-car vehicle model, see Fig. 1, whose response at speed 80 km/h is accumulated to yield a roughness index with units of slope (in/mi, m/km, etc.). More precisely, IRI is defined as the accumulated suspension motion divided by the distance travelled. The parameters of the quarter vehicle are defined by the so-called Golden Car with parameters given in Fig. 1. Since its introduction in 1986, IRI has become the road roughness index most commonly used worldwide for evaluating and managing road systems.



Golden Car		
Symbol	Value	Unit
$c = c_s/m_s$	6.0	s^{-1}
$k_1 = k_t/m_s$	653	s^{-2}
$k_2 = k_s/m_s$	63.3	s^{-2}
$\mu = m_u/m_s$	0.15	-

Quarter Truck		
Symbol	Value	Unit
m_s	3 400	kg
k_s	270 000	N/m
c_s	6 000	Ns/m
m_t	350	kg
k_t	950 000	N/m
c_t	300	Ns/m

Fig. 1: Quarter vehicle model.

Assuming a Gaussian model for road profile, the expected IRI can be computed as

$$IRI = A(w, v) \cdot \sqrt{C} \tag{2}$$

where $A(w, v)$ is a constant depending on the waviness w and the speed v. The theoretically derived relationship between IRI and C was established by Sun et al. [14], see also Johannesson & Rychlik [9], and agrees with the empirically found formula by Kropáč & Múčka [10], [11]. For the Golden car, $v = 80$ km/h, and ISO spectrum with waviness $w = 2$, the formula simplifies to

$$IRI = 2.21 \cdot \sqrt{C} \tag{3}$$

where the roughness coefficient C has unit m·mm^2.

4. Fatigue Damage Index

We will here define a fatigue damage index that is assessed by studying the response of a quarter-vehicle model travelling at a constant speed on road profiles, see Fig. 1. To be more precise, the response considered is the force acting on the sprung mass. Such a simplification of a physical vehicle cannot be expected to predict loads exactly, but it will highlight the most important road characteristics as far as fatigue damage accumulation is concerned. The parameters in the model are set to mimic heavy vehicle dynamics, following Bogsjö [2]. Thus, the values of the parameters differ somewhat form the ones defining the Golden car.

The force response of the quarter-vehicle can be computed through linear filtering of the road profile. The purpose of this work is to propose models for road profiles defined by means of a few parameters that could be used to compute the vehicle response, and hence the most important criterion for a good model of a measured road profile is that the rainflow damage of the response is well represented. Thus, the fatigue damage index is defined by using rainflow cycle counting of the response in combination with the Palmgren-Miner rule, see [9] for details.

5. Laplace modelling of road profiles

Stationary Gaussian loads have been extensively studied in literature and applied as models for road roughness, see e.g. Dodds & Robson [6] for an early application. However, the authors of that paper were aware that Gaussian processes cannot "exactly reproduce the profile of a real road". In Charles [5] a non-stationary model was proposed, constructed as a sequence of independent Gaussian processes of varying standard deviations but the same standardized spectrum. Knowing durations and sizes of standard deviations the model is a non-stationary Gaussian process. Similar approaches were used in Bruscella et al. [4], and Rouillard [12], [13]. The variability of the standard deviation was modelled by a discrete distribution taking a few number of values (in published work the number of values was six). In Rouillard [13] random lengths of constant variance sections were also considered.

To summarize, the Gaussian model is frequently used for modelling road profiles. However, the Gaussian model is often only valid for short sections, say 100 m, whereas longer sections of roads, say 10 km, typically contain shorter sections with high irregularity, and the variability between sections is higher than can be explained by the stationary Gaussian model. This phenomenon can be captured by a Laplace process, which can be seen as a Gaussian process with randomly varying variance. Thus, the Gaussian process is a special case of the Laplace process. The Laplace process may be described by its PSD, skewness and kurtosis. A stationary Laplace process can be constructed as a Laplace Moving Average (LMA), where the variance of the process is continuously varying according to a so-called Laplace motion. A non-stationary Laplace process can be constructed by randomly varying the local variance for each 100-metre section according to a gamma distribution. Here we will employ the Laplace-ISO model by using the ISO road spectrum. The Laplace-ISO model can be described by only two parameters, namely its mean roughness, C and the Laplace shape parameter, ν (or equivalently by its variance and kurtosis). The parameters can be estimated from a sequence of IRI-values, which is often available, and can then be used for reconstruction of road profiles. Simulated profiles for different Laplace shape parameters are shown in Fig. 2. More details on Laplace road models are found in Bogsjö et al. [3], Johannesson & Rychlik [9], including Matlab code for simulating Laplace-ISO road profiles.

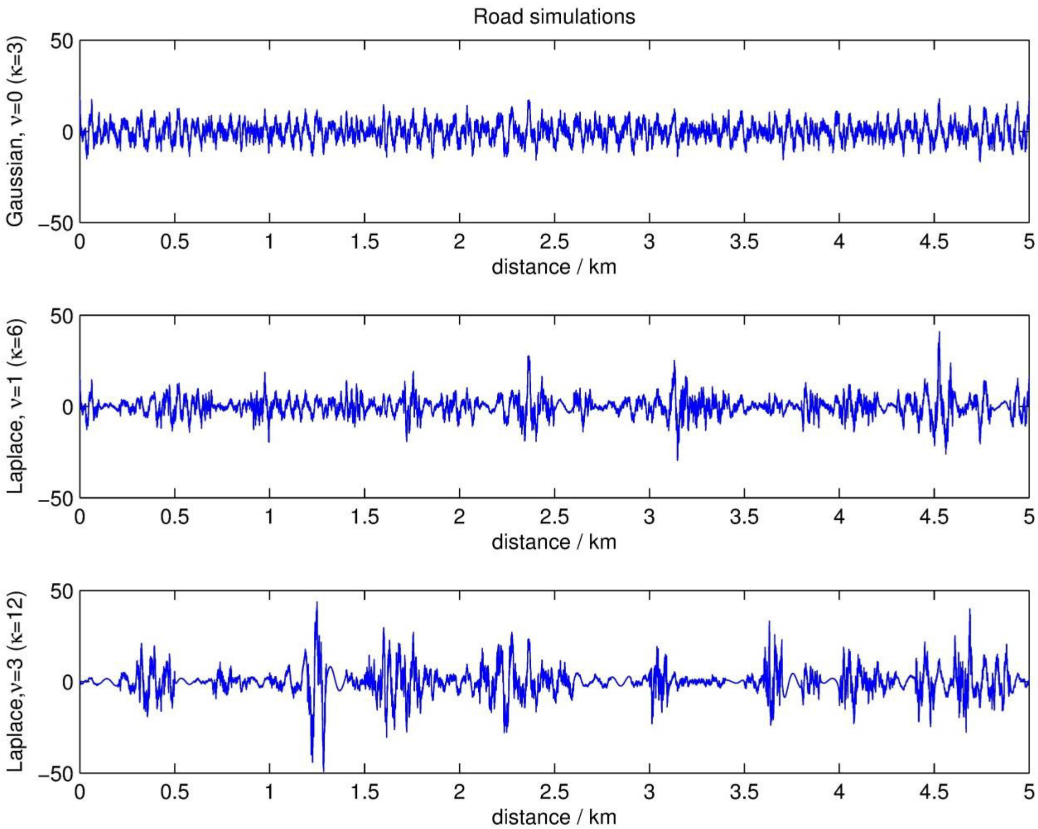


Fig. 2: Simulations of road profiles; Laplace processes with different kurtosis, κ = 3, 6, 12; κ=3·(ν+1). (Note that Laplace with κ=3 gives the stationary Gaussian process)

6. Validation of Laplace road profile modelling

An important question is how well the Laplace-ISO model fits measured road profiles. This is studied in Johannesson & Rychlik [9], and the main results and conclusion are presented below. For the validation a data set of eight sections of roads with measured road profiles were used. The eight road sections represent different types of roads as well as different geographical locations, and their lengths vary between 14 and 45 kilometres. To validate the Laplace-ISO road profile model, the following issues were studied:

1. Can the non-stationary Laplace model be used to reconstruct road profiles?
2. Can the ISO spectrum give sufficiently accurate approximations of road profiles?
3. Can the IRI be used to estimate the ISO spectrum?
4. What is the suitable length of segments with constant variance?

The accuracy of the Laplace model was validated by means of Monte Carlo studies. Relative damages, ratios between simulated and observed damage indices, were used as measures of model accuracy. A summary of the results is:

1. The general non-stationary Laplace model, with observed spectra, was used to describe the variability of the eight measured road profiles. Three factors are considered; length of the constant variance segment on three levels $L = 100{,}200{,}400$ m; damage exponent on three levels $k = 3{,}4{,}5$; and speed on two levels $v = 10, 15$ m/s. For each combination of factors and measured roads one Laplace road profile has been simulated, in total 144 road profiles. From Fig. 3 one can see that relative indices are close to one which means that damage indices computed for the non-stationary Laplace model agrees very well with the observed ones for wide range of values of the considered factors. Medians of relative damage indices are about 1.2 indicating that Laplace model is slightly more damaging than the measured profiles. We have demonstrated that the non-stationary Laplace model having observed spectrum reproduces the damage indices very well.
2. Here we investigate whether the simpler Laplace-ISO model could be used instead. The first approach is to use the Laplace parameters estimated above and simply replace the observed spectrum with the ISO one. The accuracy of this Laplace-ISO model is not good, however with conservative damage estimates. The second approach is to estimate the Laplace parameters by using two observed damage values for exponents $\mathrm{k} = 2{,}4$. Thus, parameters (C, ν) estimated in this way will define Laplace-ISO models which have expected damage indices equal to the observed damage indices for $\mathrm{k} = 2{,}4$. This estimation is based on the theoretical result on the expected damage that is described below. The resulting Laplace-ISO models are sufficiently accurate proving that 'useful' Laplace-ISO models are available for the studied roads.

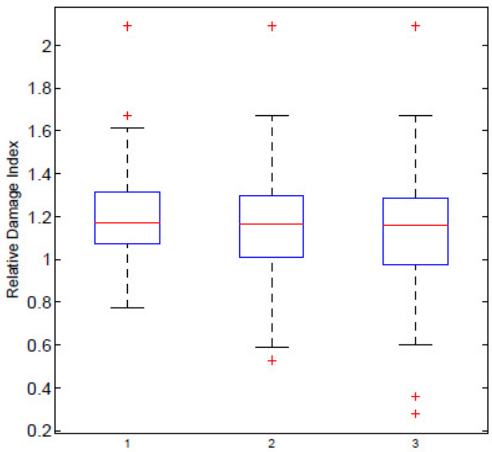


Fig. 3: Three box plots of relative damages estimated for the general non-stationary Laplace model, for damage exponents k=3,4,5, respectively. One relative index is computed for each of the eight roads and combinations of the following factors v=10, 15 m/s, L=100, 200, 400 m, i.e. each box plot is based on 48 relative indices. In a box plot, the box represents 50% of the data, while the whiskers show all data except the ones considered as outliers and marked by '+'.

3. In many countries the sequences of IRI are collected and saved in databases. Therefore, reconstruction of the road profiles from IRI sequences is of practical interest in cases when the measured profiles are not available. For validation purposes, we have estimated the sequence of IRI from the measured data and used these to estimate the parameters (C, ν) in Laplace-ISO models. We conclude that the presented approach to estimate Laplace-ISO models from IRI sequences is useful for reconstruction of road profiles when measurements are not available. In the presented validation study only models for the road profiles 6-8, which are not very damaging, give too conservative damage estimates, while for the other roads the estimated damage is almost within a factor 2.
4. Using Laplace-ISO models, estimated from the IRI sequences, road profiles were simulated for three values of parameter L (100, 200 and 500 m). One can observe that the damage varies considerably with the chosen length, demonstrating the importance of proper selection of parameter L. For high parameter values (e.g. L=500) the damage index will be smaller compared to the index for lower values (e.g. L=100). The reason is that there is a higher frequency of transients when the roughness coefficient is changing values often. A suitable value of L should be chosen based on experience and intended application, typically in the range of one or some hundred metres. For the Finnish data below $\mathrm{L} = 100$ m have been used.

Thus, the conclusions are that the non-stationary Laplace model works well for modelling of road profiles; the Laplace-ISO model, with only two parameters, is sufficiently accurate when the model is estimated either from observed damages or from IRI sequences, and; a suitable length of stationary segments is in the range of one or some hundred metres. More details on the validation are found in Johannesson & Rychlik [9].

7. Laplace modelling of roads in Finland

We here present results on Laplace modelling of the Finnish roads, where we have IRI values for 100 metre road segments. The data has been obtained from the Finnish Transport Agency. In total there are 5391 road sections longer than 5 km. Most of the road sections are between 5 and 10 kilometres.

The goal here is to describe and classify roads according to the Laplace parameters and relate these to functional class, pavement class and other road attributes. Thus, for each longer section (at least 5 km) of the roads, Laplace models have been estimated based on the sequence of IRI-values. First the C-values for 100 metre segment are estimated from IRI-values

$$C_i = \left(\frac{IRI}{2.21}\right)^2 \tag{4}$$

by using Eq. (*3*). In the second step the Laplace parameters are estimated, according to

$$C = \frac{1}{M}\sum_{i=1}^{M} C_i \quad , \quad \nu = \frac{\frac{1}{M-1}\sum_{i=1}^{M}(C_i - C)^2}{C^2} \tag{5}$$

i.e. the roughness coefficient C is the mean roughness, and the Laplace shape parameter ν is the coefficient of variation squared, see [9] for details. Once the Laplace parameters have been estimated from IRI-values, it is possible to stochastically reconstruct a road profile, see Fig. 2 for examples. However, here we will investigate the variability of the Laplace parameters, and thus Box plots of the estimated parameters are presented in Fig. 4 for different road classes, see Tab. 1.

Tab. 1: Road classes of Finnish roads.

Functional class	Number of sections	Pavement class	Number of sections
1 = main road	1131	1 = asphalt concrete	2561
2 = main road 2nd class	577	2 = soft asphalt concrete	2732
3 = local road	1479	3 = surface treatment of a gravel road	98
4 = connecting road	2204		

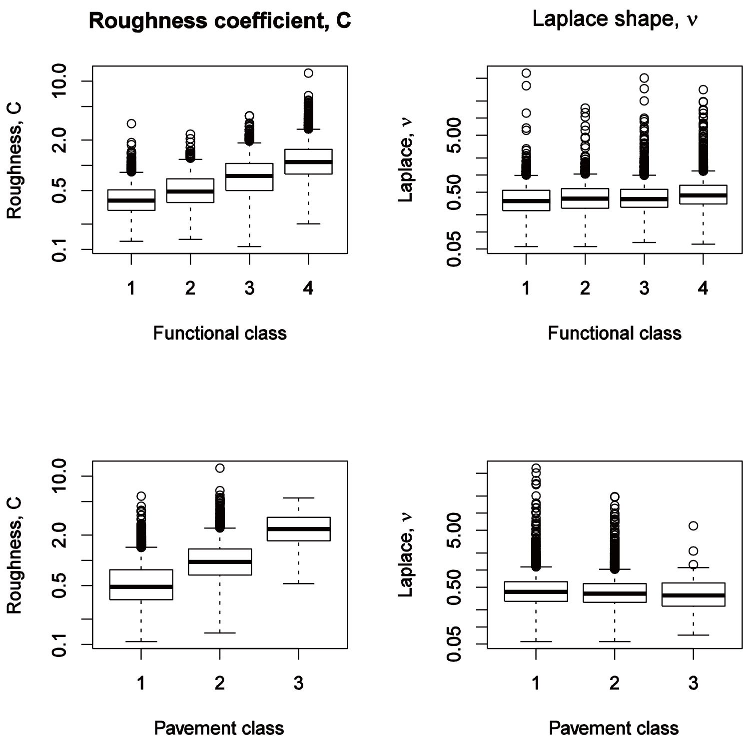


Fig. 4: Estimated parameters (C, ν) of Laplace models for roads in Finland.

In Fig. 4 we can observe that there is high variability in the estimates. However, it can be clearly seen that the roughness C increases both with functional class and with pavement class, which is what can be expected. The Laplace parameter ν is harder to interpret, and here we can see no clear tendencies, but rather that the Laplace shape parameter seems to be independent of the pavement and functional class.

We now investigate the distribution of the Laplace parameters within a certain functional class by making normal probability plots. As an illustration, the graphs for pavement classes 1 and 2 are presented in Fig. 5, where the logarithmic Laplace parameters are analyzed. It can be seen that for functional class 1, the normal distribution fits well in the central part but not in the tails. For functional class 2 the fit is excellent for the roughness, but not for the Laplace parameter. The same behaviour is found for classes 3 and 4. The conclusion is that the log-normal distribution fits well to the roughness. For the Laplace shape parameter the log-normal predicts too low values in the right tail, but otherwise the fit is good.

It seems adequate to characterize the distribution of the logarithmic Laplace parameters by their mean and standard deviation, see Tab. 2. Here we present the median of the parameter, instead of the logarithmic mean, calculated as: $\mathrm{median}(C) = \exp(\overline{\ln C})$.

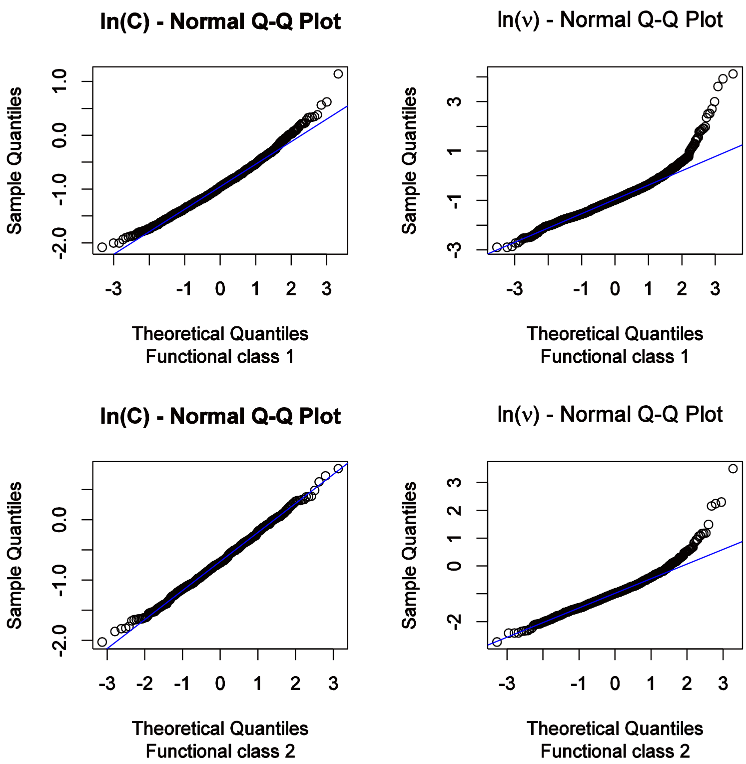


Fig. 5: Normal probability plots of estimated parameters of Laplace models for roads in Finland.

Tab. 2: Distribution of Laplace parameters for different functional classes.

Functional	Roughness, C		Laplace, ν	
class	median	std-log	median	std-log
1	0,39	0,43	0,37	0,65
2	0,50	0,47	0,41	0,71
3	0,73	0,54	0,41	0,66
4	1,12	0,54	0,47	0,67

As was concluded before, the median roughness increases with the functional class, while the median of the Laplace shape parameter does not seem to depend on the functional class and the median Laplace parameter is about 0.4 (i.e. slightly non-Gaussian). The standard deviations in log-scale can be interpreted as the relative scatter, and it does not seem to depend on the functional class. For the roughness it is about 0.5 and for the Laplace parameter it is about 0.7. The conclusion is that, for the Finnish roads, only the median roughness depends on the functional class, while the median Laplace parameter and the relative scatters do not depend on the functional class.

We have demonstrated that for a certain functional class, the Laplace parameters may be characterized by their medians and relative scatters, where only the median roughness varies between different classes for the Finnish roads. This characterization of road types can be useful for reconstruction of roughness parameters for roads in regions where there are no roughness data available. In such a case the roughness parameters could be reconstructed based on, for example, road type (e.g. functional class) and general standard of region (e.g. obtained from socio economic data).

8. Expected damage of Laplace road profiles

A practically important theoretical result on the expected damage for the Laplace-ISO model is presented in Johannesson & Rychlik [9]. The expected damage index, with damage exponent k, for a Laplace road with ISO spectrum and Laplace parameters (C, ν) can be approximated by an explicit algebraic expression

$$D_\nu(k, C, v) = 0.07093 e^{13.92k} \left(\frac{C}{C_0}\right)^{\frac{k}{2}} \left(\frac{v}{v_0}\right)^{\frac{k}{2}-1} \nu^{\frac{k}{2}} \frac{\Gamma\left(\frac{k}{2}+\frac{1}{\nu}\right)}{\Gamma\left(\frac{1}{\nu}\right)} \tag{6}$$

where the reference values are $C_0 = 14.4\ \mathrm{m}^3$ and $v_0 = 10\ \mathrm{m/s}$. The first factor depends on the parameters of the quarter-vehicle response, and the coefficients have been estimated from a very long simulation of a Gaussian road. The coefficients 0.07093 and 13.92 will be different for another filter. The second factor is a correction for the average roughness C. The third factor is a correction for the vehicle speed v. The last factor is a correction for the Laplace model, depending only on the Laplace shape parameter ν.

As an example, we have computed the expected damages under Gaussian and Laplace model assumptions for the Finnish roads, see Fig. 6. Damage exponent $k = 5$ and vehicle speed 80 km/h were used in the damage calculations. We can see that the Laplace model predicts higher damages compared to the Gaussian model, which is due to the correction factor depending on the Laplace shape parameter ν.

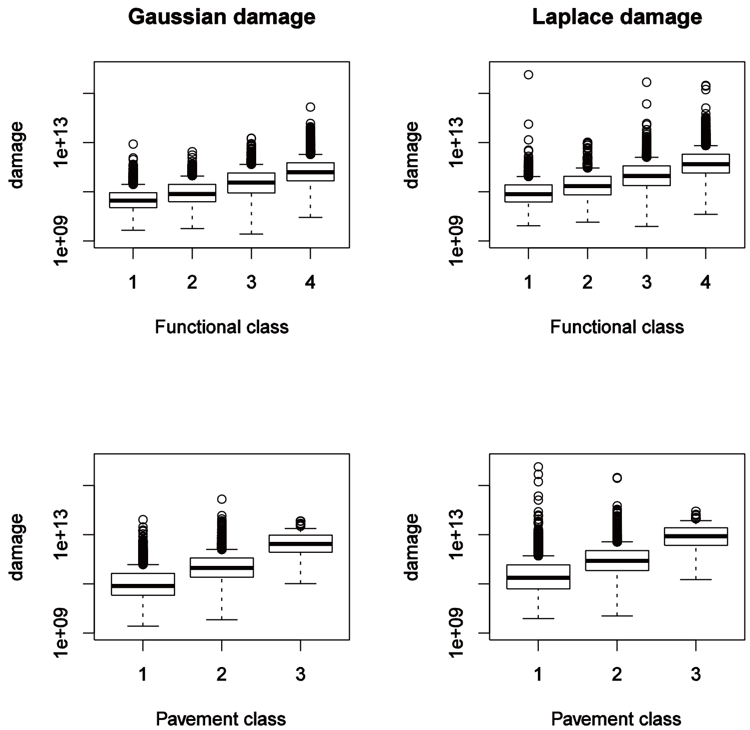


Fig. 6: Theoretically computed expected damages for Gaussian and Laplace models.

9. Conclusions

The main goal has been to find a statistical model for road profiles, which can be estimated from a sequence of IRI measurements. The road profile can then be stochastically reconstructed. When measured road profiles are not available, but only condensed roughness data in the form of IRI values or roughness coefficients, a simple statistical model for the road profile is needed in order to be able to estimate the model parameters. However, the model should still be useful for durability applications. For this purpose, the Gaussian model has been found to be too simple, see e.g. Bogsjö [2], since it cannot correctly capture the variability of the roughness. For our setup we have found that the Laplace-ISO model, i.e. a non-stationary Laplace model, Bogsjö et al. [3], with ISO spectrum [8], is simple enough but still useful for durability evaluations, see Johannesson & Rychlik [9]. It can be interpreted as a Gaussian process where the local variance is randomly varying according to a gamma distribution. The length of constant variance segments is predefined, and for road profiles typically one or some hundred metres. The Laplace-ISO can be modelled by two parameters; its mean roughness and Laplace shape parameter. We found that the presented approach to estimate Laplace-ISO models from IRI sequences is useful for reconstruction of road profiles when profile measurements are not available. There are several advantages to use the Laplace road profile model with ISO spectrum

- a small number of parameters are needed to define the model (the roughness coefficient, C, the Laplace shape parameter, ν, and the length of constant variance road segment, L),
- the parameters C and ν can be estimated from the sequence of IRI, which is often available, and can then be used for reconstruction of road profiles, and
- the expected damage of a response of a vehicle, modelled by a linear filter, having Laplace-ISO road as an input, can be accurately approximated by an explicit formula depending only on the Laplace parameters, the damage exponent and the speed.

The last property is particularly convenient for sensitivity studies since lengthy simulations can be avoided. It can also be used for estimation of Laplace parameters and for classification purposes. The usefulness of the Laplace-ISO model was demonstrated for Finnish roads, where it was found that only the mean roughness varies between different road types, while no effect on the Laplace shape parameter was found.

Acknowledgements

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