Assessing the Uncertainty in Projecting Local Mean Sea Level from Global Temperature

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ABSTRACT

The process of moving from an ensemble of global climate model temperature projections to local sea level projections requires several steps. Sea level was estimated in Olympia, Washington (a city that is very concerned with sea level rise because parts of downtown are barely above mean highest high tide), by relating global mean temperature to global sea level; relating global sea level to sea levels at Seattle, Washington; and finally relating Seattle to Olympia. There has long been a realization that accurate assessment of the precision of projections is needed for science-based policy decisions. When a string of statistical and/or deterministic models is connected, the uncertainty of each individual model needs to be accounted for. Here the uncertainty is quantified for each model in the described system and the total uncertainty is assessed in a cascading effect throughout the system. The projected sea level rise over time and its total estimated uncertainty are visualized simultaneously for the years 2000–2100, the increased uncertainty due to each of the component models at a particular projection year is identified, and estimates of the time at which a certain sea level rise will first be reached are made.

1. Introduction

The city of Olympia, which is the capital of Washington and is located at the southern end of Puget Sound about 100 km south of Seattle, is facing an increasing threat of sea level rise. There is disagreement about how quickly it will happen and how drastic it will be. Planning for a 15-cm rise is very different from planning for an 80-cm rise. It may take years to get approval for any adaptation effort, such as building a sea wall, and thus planning must start early (Craig 1993). We aim to provide tools to aid planning efforts by recognizing that point estimates of future sea level rise do not capture the full range of possible outcomes. As we will discuss, under different

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climate scenarios there are ranges of both timing and estimated sea level that will occur in Olympia with given confidence. Although the data used in our analysis are focused to a particular region of the United States, the same underlying strategy to identify confidence ranges around expected time frames and their associated sea level remains transferable as a sound tool for decision making. The financial impact of different scenarios of sea level rise can be assessed using the methods of Hallegatte et al. (2013), who estimate the vulnerability of a city by comparing its annual average loss from flooding with its annual gross domestic product. Using our approach, one can translate the distribution of projected sea level increase into a distribution of annual financial loss and hence of vulnerability, rather than focusing on averages. This approach can, in turn, be used to assess the likely effect of and need for investments in adaptation measures.

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Systematic quantification of uncertainty gives policymakers a better idea of what to prepare for in the future (Reilly et al. 2001; Katz 2002; Stephenson et al. 2012). Instead of point estimates (often without uncertainty measures; e.g., Mote et al. 2008) for each scenario, we calculate simultaneous confidence sets of sea level projections. These are realistic benchmarks for when preventive measures need to be taken against the threat of sea level rise and how long one can afford to wait before taking action. For example, in an Olympia city report (Simpson 2011), various types of sea walls were proposed in response to different rises in mean sea level. To plan adequately for building these walls, our projections give policymakers a range of years for when they can expect certain levels of sea level rise. Building the sea walls in the earlier years of this range would be advisable, since the projections are intended to help the city avoid excessive flooding. Such flooding typically occurs at spring tide. In 1978 the city encountered a high tide of over $5.5 \,\mathrm{m}$ (the mean sea level is $2.5 \,\mathrm{m}$), and the range of tides (from lowest low to highest high) can reach 6.8 m (see online at http://tidesandcurrents.noaa.gov/benchmarks/ 9446969.html). Thus, even a small increase in mean sea level is likely to change the characteristics of flooding, such as the design-life level (Rootzén and Katz 2013) of a sea wall.

2. Data

In our analysis we use 18 climate models from phase 5 of the Coupled Model Intercomparison Project (CMIP5; Taylor et al. 2012) that project global mean temperatures for each of the four representative concentration pathways (RCPs; Moss et al. 2010) used in the Fifth Assessment Report of the Intergovernmental Panel on Climate Change (IPCC; Stocker et al. 2013) (at the time of writing, information on the climate models used could be found online at http://courses.washington.edu/statclim/ GCMs_used.html). The projected global mean temperatures for each model in this ensemble are then used to project global sea level using a relationship [Eq. (1), below], fitted to historical data: the Goddard Institute for Space Studies global annual mean temperature series (Hansen et al. 2001) and global sea level data from the Commonwealth Scientific and Industrial Research Organisation (Church and White 2011). The former is computed from archives of land and sea air temperature measurements, and the latter is based on tide gauge readings using a spatial structure determined by satellite data. Using a long time series of annual tide gauge data for Seattle from the National Oceanic and Atmospheric Administration (Zervas 2009), we relate the global sea level to Seattle sea level, and, because Olympia does not have a long record of reliable sea level data, we use tide gauge calibration data (http://tidesandcurrents.noaa.gov/ inventory.html?id=9446807) for Budd Inlet (the southern part of Puget Sound where Olympia is located) to relate Olympia to Seattle. [All data used are available (as links in the R software code file) at http:// www.statmos.washington.edu/datacode.html.]

3. Methods

a. Predicting global mean sea level from global mean temperature

Rahmstorf (2007) proposed to relate global sea level change dH_t/dt to temperature T_t using the equation

$$dH_t/dt = a(T_t - T_0) + \varepsilon_t, \qquad (1)$$

where a and T_0 are parameters and ε_t is a noise series. We assume that the noise series is a moving-average process of order 2 (Box and Jenkins 1970), as determined by the R function "auto.arima" in the "forecast" package (Hyndman et al. 2013). Different from Rahmstorf's statistical approaches (Rahmstorf 2007; Vermeer and Rahmstorf 2009), we do not smooth the series (because smoothing just adds complexity to the time series structure), do not add a term corresponding to dT_t/dt (because it is not statistically significant), and do not make a reservoir correction. The details of these choices are given in Guttorp et al. (2014). Figure 1a shows the sea level data (Church and White 2011) and the fitted model, with the corresponding residual plot given in Fig. 1b. The estimated values are $\hat{a} = 0.18 \text{ cm} (\text{yr} \circ \text{C})^{-1}$ (standard error of 0.05) and $\hat{T}_0 = -1.12$ °C (standard error of 0.26). All computations are made using the freely available statistical software package R (R Core Team 2012; the code used is available at http://www.statmos.washington.edu/ datacode.html).

b. Predicting Seattle sea level from global sea level

Figure 1c and the residuals in Fig. 1d indicate the adequacy of a linear model, fitting Seattle sea level data Y_t to global sea level data H_t , namely,

$$Y_t = \alpha + \beta H_t + \zeta_t, \qquad (2)$$

where ζ_t is an error term with temporal structure a moving average of order 1, again determined using the R "forecast" package (Hyndman et al. 2013). The estimated parameters are $\hat{\alpha} = -0.96$ cm (standard error of 0.51) and $\hat{\beta} = 1.23$ (standard error of 0.07), and so Seattle sea level somewhat attenuates global sea level. According to the National Research Council (NRC 2012), processes that raise relative sea level in the northeastern



FIG. 1. (a) Observed and fitted global sea level anomalies and (b) the corresponding residual plot. (c) Seattle sea level anomalies plotted against global sea level anomalies for 1899–2009 with fitted line and (d) its residual plot. (e) Budd Inlet sea level plotted against Seattle sea level for calibration data in 1996 with fitted line and (f) the residual plot. Note that the time scale for (f) is days rather than years.

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Pacific Ocean include warm phases of climate oscillations and land subsidence due to glacial isostatic adjustment, sediment compaction, and the withdrawal of groundwater, whereas relative sea level is lowered by cool phases of climate oscillations, gravitational and deformational effects of modern melting of glaciated landmasses, and land uplift due to tectonics or fluid recharge. A detailed analysis of these effects is made in NRC (2012) but has not been attempted here, since their uncertainties appear to be of smaller order of magnitude than those involved in our time series regressions.

c. Predicting Olympia sea level from Seattle sea level

Figure 1e shows 9 months of daily calibration data relating daily mean sea level Z_t at Budd Inlet (Olympia) to the Seattle sea level data, and Fig. 1f depicts the residuals from the model given in Eq. (3). According to the U.S. Geological Survey calibration sheets, the difference in mean sea level between Seattle and Budd Inlet is 51.3 cm. A shift model relating the two series is

$$Z_t = Y_t + \gamma + \eta_t, \tag{3}$$

where η_t is an autoregressive moving-average (ARMA) (2, 2) error term, again determined using the "forecast" package in R. The estimated parameter is $\hat{\gamma} = 51.1$ cm (standard error of 0.19), in good agreement with the benchmark value. Once we correct for the average sea level from 1970 to 1999, the shift disappears and the uncertainty due to the shift is just the very small standard error of $\hat{\gamma}$. The relationship between sea levels at Olympia and Seattle is very simple, since they are part of the same sound with no ocean inlets between them.

d. Propagation of variability

To project future sea levels in southern Puget Sound, we employ an ensemble of climate models from CMIP5 (Taylor et al. 2012). The climate model projections are used to get an idea of the variability due to model uncertainty. We then apply the fitted regression Eq. (1) to the temperature projections. Last, we use the joint distribution of the two downscaling estimates (to Seattle and to Olympia) and apply the downscaling Eqs. (2) and (3) to the estimated global sea level projections. When estimating confidence range, it is important to take all of the known uncertainties into account. Each prediction step above increases the uncertainty and widens the range. The final step is to use the work of Bolin and Lindgren (2014) to widen the pointwise intervals so as to make them simultaneous. This step enables us to produce valid projection intervals for the timing of a particular amount of sea level rise as well as estimating the probability of not exceeding a particular level before a given year. We use 90% confidence level to conform to the usage in the latest IPCC report (Stocker et al. 2013).

e. Simultaneous confidence regions

The joint distribution of the Olympia sea level $\mathbf{Z} = \{Z_{2000}, \ldots, Z_{2100}\}$ in the period 2000–2100, conditionally on the climate model outputs, the past sea levels, and the estimated model parameters, is a mixture of K = 18Gaussian distributions (Bolin 2012), corresponding to each of the climate models used:

$$\pi(\mathbf{Z}) = \sum_{k=1}^{K} \frac{1}{K} \frac{|\mathbf{Q}|^{1/2}}{(2\pi)^{101/2}} \exp\left[-\frac{1}{2}(\mathbf{Z} - \boldsymbol{\mu}_k)^{\mathrm{T}} \mathbf{Q}(\mathbf{Z} - \boldsymbol{\mu}_k)\right].$$
(4)

Here, μ_k is determined by climate model k and the common precision matrix **Q** is determined by the stochastic moving-average models, the uncertainty in the model in Eq. (3), and the uncertainty in the lower integration bound in the integrated version of Eq. (1). A pointwise confidence band is constructed by finding the interval $[q_{0.05}(t), q_{0.95}(t)]$, for each time t, where $q_{\alpha}(t)$ denotes the α quantile of the marginal distribution $\pi(Z_t)$.

A simultaneous confidence region, such that, with probability $1 - \alpha$, the process stays inside the band at all times $t \in [2000, 2100]$, can be constructed by considering the joint distribution for **Z**. We construct this simultaneous band by finding the value of ρ such that

$$P[q_{\rho}(t) < Z_t < q_{1-\rho}(t), 2000 < t < 2100] = 1 - \alpha.$$
 (5)

Finding ρ requires that we can calculate probabilities $P(\mathbf{a} < \mathbf{Z} < \mathbf{b})$ efficiently. This is done using the sequential integration method by Bolin and Lindgren (2014), implemented in the R package "excursions." Finding the probability of staying below a given level *u* until a particular year *T* requires calculating probabilities $P(Z_t < u, 2000 < t < T)$, which also is done using the excursions package. More details about the procedure can be found in Guttorp et al. (2014).

4. Results

Figure 2 shows a simultaneous 90% confidence region for the sea level projections and the projections based on each of the 18 climate models. There is considerable uncertainty (due to local and global statistical model fitting) beyond the ensemble variability. The pointwise confidence intervals (dashed purple lines) are only correct when looking at sea level at a given individual year, whereas the simultaneous confidence intervals (black



FIG. 2. Olympia sea level projection simultaneous 90% confidence set (thick black lines) for the years 2000–2100 for four climate scenarios. The sea level data are shown in blue and end in 2009, and the average projections are the thick dark blue lines. The thin red lines are the projections without uncertainty that are based on each of the climate models. The dashed purple lines connect pointwise confidence intervals for each year.

lines) are valid for all years at the same time, giving policymakers the tools to make informed decisions on phenomena that may span multiple years without having to perform any additional calculations.

Looking across the confidence set vertically at a given year (here 2075), we can visualize the additional contribution of each part of the chain of models to the overall uncertainty (Fig. 3). There is very little additional uncertainty added in going from Seattle to Olympia, and therefore the blue and purple lines are almost on top of each other.

Looking across the bands in the horizontal direction we obtain a 90% confidence interval for the time when Olympia may see a given level of sea level rise over the



FIG. 3. 2075 Olympia sea level projections with uncertainty due to different sources for four climate scenarios. The black line is the median projection (with no uncertainty), and the histogram corresponds to the spread of the climate models. The red curve adds the uncertainty due to the relation between global temperature and global sea level, the blue line adds the additional uncertainty due to downscaling global sea level to Seattle, and the purple line adds the additional uncertainty due to the projection of Budd Inlet (Olympia) sea level. The purple lines are on top of the blue lines almost everywhere.

1970–99 average sea level. As an example, Table 1 shows high and low estimates together with the mean firstoccurrence year for each of the RCPs and two different levels. For example, a 50-cm rise is reached earlier under RCP 8.5 than for the other scenarios.

Figure 4 shows the probability of staying below a given level until a particular year. The probability of not exceeding 25 cm is hardly affected by the RCP used, whereas there are substantial differences between RCPs for staying below 50-cm sea level rise. The city planners in Olympia have agreed to plan for the high end (Mulkern 2013), which in our analysis would correspond to RCP 8.5. The price of building a sea wall needs to be weighed against the potential price of the cleanups due to the flooding that a 50-cm mean sea level rise (in conjunction with a storm surge, spring tide, and low pressure) could cause. Similar computations can easily be done for any sea level rise of interest.

5. Discussion

The estimation of variability in the model projections using an ensemble may not be all that accurate, since climate models, particularly from the same modeling

TABLE 1. Projected 90% confidence intervals for reaching particular mean sea level rise in comparison with 1970–99 averages.

Level (cm)	Earliest year	Mean year	Latest year
	RCP	2.6	
25	2029	2051	2083
50	2068	2099	After 2100
	RCP	4.5	
25	2029	2050	2074
50	2065	2088	After 2100
	RCP	6.0	
25	2029	2051	2075
50	2062	2088	After 2100
	RCP	8.5	
25	2029	2048	2067
50	2062	2078	2097

group, tend to be statistically dependent (Jun et al. 2008; Knutti et al. 2010). Furthermore, the selection of models submitted to CMIP5 is perhaps not representative of all modeling efforts. Both of these effects may lead to a biased estimate of the variability. Because there is no obvious approach that enables more accurate estimation of the model variability, we will just argue conditionally upon the projections.

In this work we use frequentist methods, but similar calculations can be done using Bayesian analyses. In either case, the approach uses a hierarchical model, which is the predominant statistical approach to analyze complicated systems (Katz et al. 2013). One could extend the uncertainty analysis by including historical climate simulations in the modeling of global sea level

P(Y(s)<25,s<t)

from global mean temperature as done in Bhat et al. (2011). To apply the method to other cities, one just needs to develop a model that relates global sea level to the particular local sea level (Tebaldi et al. 2012). This is easiest when tide data are available at the particular location but may need a spatiotemporal prediction model at locations at which no such data have been collected.

In this paper, we have used CMIP5 temperature projections together with a semiempirical model relating historical global mean sea level to global mean temperature. Our method can be applied to any projection of global mean sea level, such as a combination of steric sea level rise from climate models and glacial models driven by climate model temperature projections as used in Stocker et al. (2013); see also Moore et al. (2013) and Orlić and Pasarić (2013) for comparisons between semiempirical and process-based projections and Church et al. (2013) for an evaluation of process-based projections. The uncertainty analysis in the process-based case would require, at a minimum, a sensitivity analysis of the glacial models. The resolution of global models is insufficient to project local sea level rise, and we are not aware of any dynamic downscaling approaches for local sea levels. Hence, regardless of how global sea level is projected, a statistical approach to downscaling appears to be necessary.

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P(Y(s)<50,s<t)



FIG. 4. Projected probability of Olympia sea level rise staying below a given level [(left) 25 cm; (right) 50 cm] for four climate scenarios. The abscissa corresponds to the quantity *t* in the header.

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