The Kleinian Lesson Planning Cycle

Upper secondary school teachers and university staff designing mathematics lessons in collaboration.

Introduction

We describe the Kleinian Lesson Planning Cycle and the results from the Swedish Klein Conference. The process intertwine university teachers advanced standpoint on mathematics and upper secondary school teachers advanced standpoint on school teaching to collaboratively design mathematics lessons in the line of Klein´s famous book “Elementary mathematics from an Advanced Standpoint”.

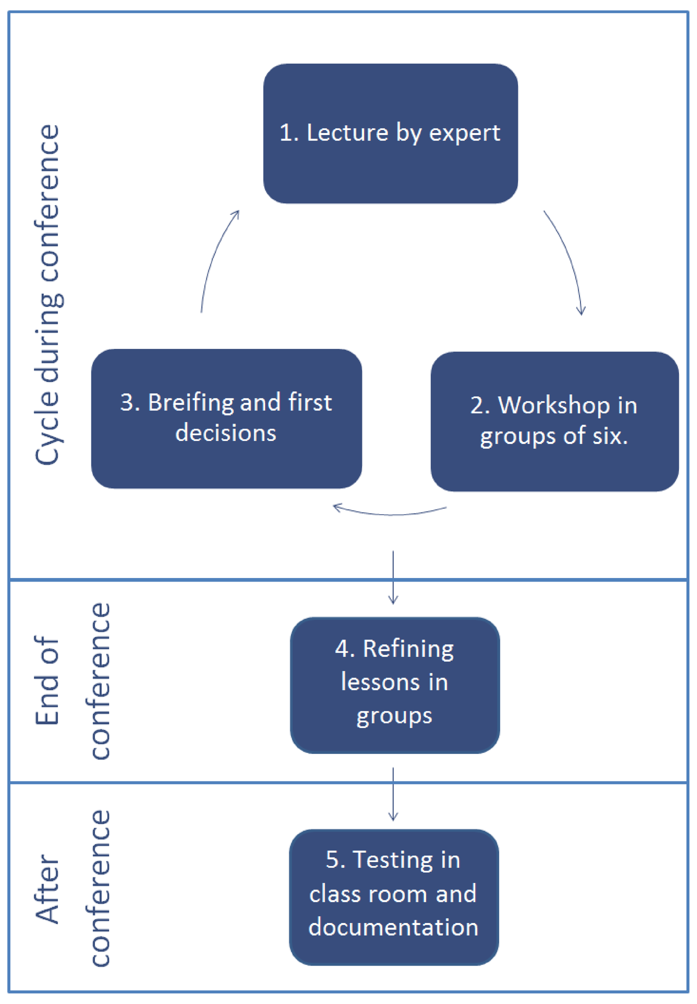
Process

Step 1: Lecture by mathematician

An expert gives a lecture knowing that the lecture will be the starting point of the lesson planning. The lecture can either focus on core content that is in the curriculum of upper secondary school, or open up for possibilities to building generic mathematical competences that are to be developed in upper secondary school.

Step 2: Workshop

The participant work in groups of six, school- and university staff mixed, to come up with ideas that can be used as parts of lessons. The groups are encouraged to consider using the 5E-model, Engage, Explore, Explain, Elaborate and Evaluate.



Step 3: Breifing

The groups shortly present their ideas in plenum. All participants can contribute to ideas displayed. A discussion, lead by a pair of so called Lesson Pilots consisting of an appointed university teacher and an upper secondary school teacher. The best ideas are identified and one discusses how these ideas can be joined to one or more coherent lessons.

Step 4: Refining

After possibly several iterations of step 1 to 3 on different mathematical themes, each iteration taking half a day, the participant are divided into groups according to interest. Each group develops a lesson sketched in step 3. The work of each group is lead by the two Lesson Pilots responsible for the lesson.

Step 5: Testing

After the conference the Lesson Pilots test the lesson in an upper secondary school class. Preferably the test contains two or more iterations with further development in between.

Developed for the Klein Conference

Background

The Kleinian Lesson Planning Cycle is developed for the Swedish Klein Conference which is part of ICMI´s Klein Project. The annual conference is organized by ICMI-Sweden in collaboration with NCM, the Swedish National Center for Mathematics Education, and Institute Mittag-Leffler, the worlds oldest mathematical research institute.

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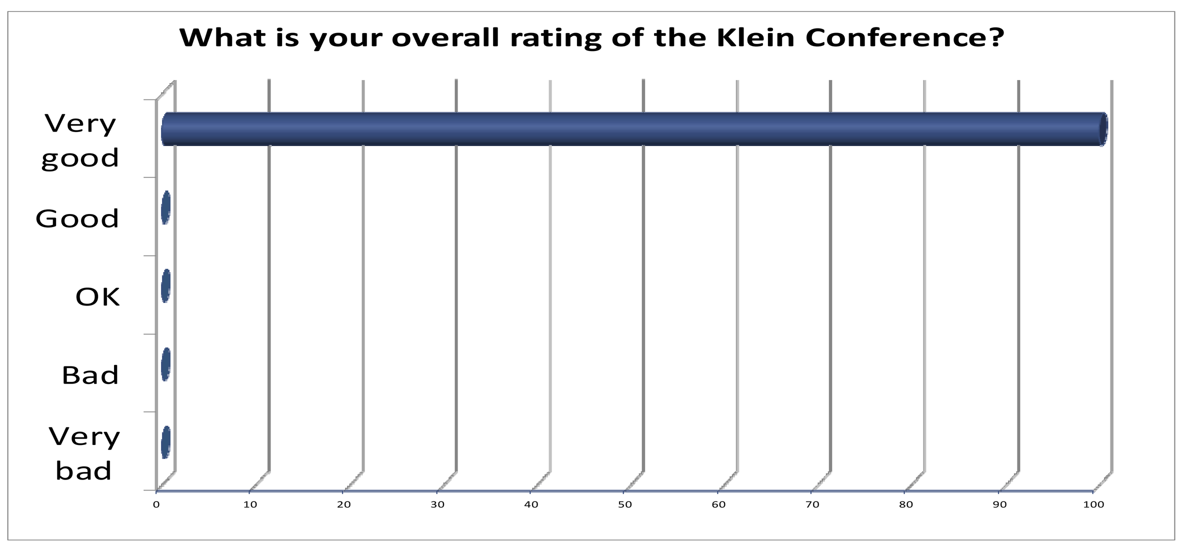
The Klein conference in Sweden gathers 20 selected upper secondary school teacher and 10 university teachers for three days. The conference results in lessons on four themes, developed in four cycles, which we call the Kleinian Lesson Planning Cycle.

> ”We worked with the most important thing for teachers, lesson planning, in an optimal way.”

Evaluation

The Swedish Klein Conference is evaluated through an electronic questionnaire. The response shows that the both the upper secondary school teachers, as well as the university teachers, find that process very fruitful.



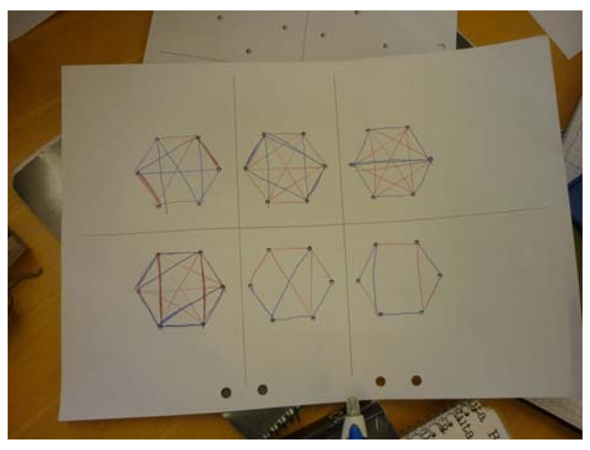
Results

Ramsey numbers

– a short description of a lesson developed

Explore

Let the students, in groups, color edges in complete graphs with first 4, 5 and the 6 nodes using a red and a blue pen. The group ”wins” if they can color all edges without getting a blue or red triangle, i.e. avoiding monochromatic complete subgraph with three nodes. The goal in this part is that the students shall detect that it is possible to ”win” in the cases of 4 and 5 nodes, but not in the case of 6 nodes.



Explain

Look at the case of complete graph with 6 nodes. In a hole-class discussion argue that, by choosing one of the six nodes, the Pigeon hole principle says that at least three of the five edges from the chosen node has to have the same color, lets say blue. The three nodes, in the other end of these three edges, are also interconnected by edges. Non of these edges can be blue since this would give a blue triangle together with the edges from the chosen node. Hence they must all three bee red, but this then gives a red triangle. Hence no coloring of these edges can avoid a monochromatic triangle.

Elaborate

We have shown that the R(3)=6, i.e. the smallest complete graph, where one can not avoid a monochromatic complete subgraphs with 3 nodes, is 6.

It is known that R(4)=18, i.e. the smallest complete graph, where one can not avoid monochromatic complete subgraphs with 4 nodes, is 18. Direct checking is impossible since even the fastest computer would use trillion times the estimated age of the universe to try out all 2^{153} possiblilites.

Leave the students fascinated by the fact that R(5) is not known, but is known to be between 43 and 49.

> ”I wish that we in upper secondary school always could plan our teaching according to the model from the Klein Conference.”

Conclusions

The Kleinian Lesson Planning Cycle has proved to be a effective process for linking together mathematical expertise with knowledge about school teaching. It involves mathematicians and school teachers in a process to develop mathematics lessons for school. The evaluation shows that the school teachers see a big advantage in being able to collaborate with mathematicians and that this supports the development of their own mathematical knowledge while constructing teaching material.

Kleinian Lesson Planning Cycle has been developed, used and evaluated for the Swedish Klein Conference. It has also been used at other national conferences and at regional school development projects.

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